

Development Optimization Model of a Zero-Defect Single Sampling plan

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Abstract

One way to check the products quality is to inspection the lot inputs. The focus of this paper is on a model of non-linear integer programming for determining an optimal single sampling plan for inspecting different parts so that the total quality control cost is minimized and we try to improve the quality of inputs to the assembly line by applying a rectifying inspection policy. The optimization model includes the inspection cost, the cost of non-conforming items entering the assembly line and the cost of rejecting the items. In this research, it is assumed that the inspection is perfect and zero acceptance number policy is employed for inspection. If a non-conforming item is found in the sample, the total lot is rejected. Each part is different in the risk of non-conforming items, the cost of non-conforming items, the size of the lot and the inspection cost. In the practical example, it can be seen that the rate of items that is defective, followed by the defective items cost and the cost of lot rejection, have been greatly reduced following the proposed methods and minimized the cost of quality control.

Keywords

Acceptance sampling; Non-linear integer programming; Incoming inspection; rectifying inspection.

1. Introduction

The complexity of the products that are being produced today has increased dramatically, and consequently quality control of the products is more difficult, and the customer demands a durable product, as a result, quality is an important aspect in product development and is a key factor in achieving target markets and gaining competitive advantages. In order to assure the quality of product, there are various tools; one of these tools is to apply sampling for acceptance, so that after collecting a random sample from the lot, the selected quality characteristic is inspected, then according to the information obtained from this inspection, decision is made on the

acceptance or rejection of the lot. Accepted lots in the production line are used, and rejected lots are reworked or returned to the supplier [1].

Recently, using zero-acceptance sampling has been growing. In this method, a lot is accepted if there are not any non-conforming items in the sample; this means that the number of non-conforming items should be equal to zero. Sampling method with zero acceptance number

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emphasizes on the lack of non-conforming items and it is easier to use for manufacturers and consumers. In this method, since the acceptance number is zero, and the lot quality has a reciprocal relationship with sample size, thus its optimization is of great importance.

Determining the sample size is significant because of its effect on the sampling costs. If the lot does not inspect, then the non-conforming items cost will be incurred and if the inspection is 100%, the inspection cost will increase significantly, so the choice of the optimal sample size should be made balance between these two costs.

In order to determine the sample size, there are different methods that one of these methods is to design the economic model that determines all related qualitative costs and aims to minimize the total cost of the capability to optimize the sample size.

2. Literature Review

To have products with high quality, the product components and the relationships between them should fall within the tolerances for the quality characteristics. Therefore, the manufacturer must inspect the components before the assembling, to ensure that they are well assembled in the product. According to the standards if the faulty items enter the assembly line, the system or product will not function properly, and this will lead to the additional cost for the company. In order to minimize production costs, sampling plans are a method to prevent the usage of non-conforming components into the final product. 100% inspection is expensive and time consuming and does not guarantee acceptance without inspection of mismatched items in finished product quality; so, a research has been done to optimize sampling plans to reduce costs. We will continue to study the research performed in this field.

Most of the activities carried out in the field of sampling have been analytical; Wetherill and Chiu [2] have also specifically referred to this issue. Several articles have been written in the field of sampling that most of them are based on the articles written by Hald [3]. One of the main reasons for applying the approach of acceptance sampling is to consider the cost of the inspection methods. For the first time, Lieberman and Resnickov [4] proposed a statistical sampling approach for the inspection of incoming lot, but the economic design for acceptance sampling was presented by Bennett et al [5]. Schmidt et al [6] also introduced another model for variable sampling plan, in which a step-by-step model was developed to calculate the cost of quality. In the proposed model, as long as quality characteristic is within the acceptance region, the cost is considered to be zero, and when the quality characteristic is outside of the acceptance region, fixed costs will be incurred also. With the introduction of Taguchi's [7] loss function, there was a revolution in the vision towards quality costs.

There are several procedures to design an acceptance sampling plan. One of the important approaches in this area is to consider the risk of accepting or rejecting the lot and economic factors. In this approach, an optimal sampling plan is obtained by taking into account the cost of rejection or acceptance.

Humzic et al [8]. presented an economic model based on a zero-acceptance single sampling plan. In this paper, two models have been investigated. The first model was designed, regardless of labor costs and the second model is based on labor costs for inspection. The proposed model provides an optimal solution for the problem. Qin et al [9] also provided a non-linear integer programming

model in 2015 to determine a zero-acceptance single sampling plan for inspection of the items entering to the assembly line. This model determined the optimal sample size for different parts. In this research, a three-step solution algorithm is proposed that significantly reduces the problem-solving time.

Willemain et al [10]. Presented an economic model for the 100% inspection problem and single sampling plan with and without of inspection errors so that they used the Taguchi model to analyze the difference between the quality characteristic value and its target value. The proposed scheme is a new design due to the use of a continuous loss function for comparison study and analyzing the inspection errors effect and the greater clarity of the model in treating with the effect of the error and also the possibility of using the model in designing control charts.

Ferrell and Choker [11] proposed an economic method for determining the optimal sampling plan by taking into account consumer and producer loss functions. In their study, they designed a single acceptance sampling plan for obtaining sample sizes and optimal acceptance numbers. The Taguchi continuous loss function has been used to determine the amount of deviation from the target level in their study. They also considered the inspection error in their model.

Niaki and Fallahnezhad [12] used Bayesian inference and dynamic planning for designing a sampling plan. The proposed model has considered cost and risk function in the objective function to obtain an optimal policy. Niaki and Fallahnezhad initially modeled the problem as a dynamic planning and then minimized the cost / (1-risk) ratio to identify the optimal decision. Ultimately, they designed and defined control thresholds to make decisions easier.

Fallahnezhad and Hosseini Nasab [13] introduced a new control procedure for acceptance sampling schemes. Decisions are made on the basis of the number of faulty items in the inspected sample. The purpose of this model is to find a constant control level that minimizes total costs, including the rejection cost, inspection cost, and the non-conforming item cost. Optimization is carried out using negative binomial distribution and Poisson distribution.

Hsu and Hsu [14] provided an economic plan for determining an optimal sampling scheme in a two-stage supply chain based on the quality cost and internal failure by taking into account consumer and producer risks. They concluded that the proposed design was very sensitive to the quality of the manufacturer product.

Fallahnezhad and Niaki [15] developed an optimization model to distinguish the optimal values of the control thresholds, so that they satisfy the constraints on the probabilities of a type 1 and type 2 errors. They have used Markov model to extract the total cost of the acceptance sampling plan, including the cost of acceptance and inspection.

Fallahnezhad and Aslam [16] provided a new sampling plan for decision-making according to the cost-objective function. They used Bayesian inference to update the probability distribution function of non-conforming proportion. In addition, Bayesian inference, along with a recursive induction, has been used to estimate the expected cost of different decisions.

Li and et al [17] reviewed the military standard MIL-STD-1916. This standard is in the form of zero acceptance number. This means that if the non-conforming items are not detected then the lot will be accepted and if any non-conforming items are found, the total lot will be rejected. In this research, the author states that if there are no non-conforming items in the sample, it does not mean that the entire lot is in accordance with the specification. Military standards also need a large number

of human resources to inspect the sample with large sizes that are not feasible in practice. Champernowne's research [18] focuses on the economic outcomes of the problem using sampling plan as quality tool in the process. Champernowne has considered three states in his problem:

1. Average lot quality for testing and testing and the change between qualitative lots of average.
2. The inspection cost and its dependence on the quantity of inspection.
3. Costs associated with making a mistake to accept or making mistakes to reject any lot and dependence of this cost to the cost of quality in each lot.

Champernowne (1953) developed an economic model to distinguish whether a lot be accepted or rejected. He focused on the economic aspects of the problem. This means that when the results are within the economic range, the lot is accepted even if there were non-conforming items in the sample. Bernard [19] believes that Champernowne's assumptions are not available in practice. Bernard proposed that considering a probability distribution for the number of non-conforming items is required in order to solve the problem.

Hamaker [20] has provided three different sampling methods:

1- Sampling tables. 2- Data collection. 3- Design of sampling plans. He also presented an economic and concluded that it would be ineffective to inspect all items in the lot if there is little chance to detect non-conforming items in the lot.

Calvin, [21] state similar concerns with regard to non-conforming items. The author believes that most of the managers only look for accessing the non-conforming zero, and they do not pay attention to staying at the non-conforming level of zero. The author believes that there are various statistical methods, including control charts and acceptance sampling schemes that managers could apply for their products to remain at a non-conforming level of zero.

If lot is rejected then production may decrease, or may be stopped due to lack of parts. Salameh and Jaber [22] proposed the optimal inventory level of items that may include non-conforming items. They shown that the number of items in each order increases if the number of non-conforming item increases. On the other hand, Maddah and Jaber [23] observed that ordering a large number of items with poor quality is not always very profitable, so a reasonable balance between transportation costs and inventory costs is needed in order to achieve more profit.

Taghipour and Benjewik [24] and Taghipour and et al [25] considered the economic aspect of the sampling plan. They considered two different kinds of failures in the system: a hard failure that breaks a system and a soft failure that does not break the system but reduces the system's effectiveness. Therefore, if the failures are not soft, the system will not be efficiently implemented and will increase the system cost.

Shi and Zhou [26] provided a brief overview of different methods to improve the quality control for the processes with multiple stages. Some of the important methods are the physical method, the data-driven model and statistical process control. Physical methods need past data of the process. The data-driven model requires adequate knowledge in mathematics and statistics as well as a database for proper estimation. Data-driven models are more attractive because do not need past information to use in the process. Statistical process control has a probability of wrong alarm and based on the findings of Shi and Zhou; there is no ability to discriminate between changes in different stages.

Starbird [27] has shown that if a manufacturer uses a zero-acceptance number for the inspection of incoming items then it exacerbates the supplier to deliver the lots with zero non-conforming items. In addition, when manufacturers use the method of 100% inspection or zero acceptance number for inspection is appropriate for inspection in manufacturer.

Fernandez [29] has presented a nonlinear integer programming problem for acceptance sampling plan for defective items per unit by considering producer and consumer risks that minimizes expected surplus costs. An algorithm to set the optimal number of units for inspection is also presented. In addition, Fernandez [30] has proposed a nonlinear integer programming problem to determine a binomial sampling scheme to investigate large lot with consumer and producer risks.

Lu Cui et al. [31] have presented a new review to redesign sample size as a time-adapted sequence sampling design to determine sample size. The new approach results in optimizing the design over a wide range of design parameters.

Sommer, And Steland [32] have proposed a new sampling method as a multi-stage framework in which the accumulation is monitored at several time points and is accepted only if it goes through all stages.

Ahmadi et al. [33] have provided a Bayesian problem of forecasting future observations with an exponential distribution according to an observed sample, taking into account both the experiment total cost and the mean square error of prediction in order to distinguish the sample size.

Determining the sample size is important when a zero-acceptance number method is used. The sample size for inspection is determined by different methods, including:

1. Using standard sampling inspection tables like Dodge and Romig [28].
2. The use of acceptance criteria, like the acceptance quality level (AQL) and the lot tolerance proportion defective (LTPD) based on producer and consumer risks.
3. Developing an economic model to take all the costs associated with quality.

Among these methods, the economic model provides a better ability to optimize the sample size [9].

Sample size determination is of great importance due to its cost impact, and by obtaining the optimal sample size we can reduce the quality control costs. The proposed model provided an optimal and economical sample size considering all costs, which minimizes the quality control cost.

3. The problem formulation

When the acceptance number in the sampling plan is zero, random samples are selected from each section lot by the inspectors. If there were no non-conforming items in the sample, the lot is accepted, otherwise the total lot is rejected and rectifying inspection will be done. Inspections in different sections can be formulated as an optimization problem, which aims to reduce total costs by selecting the optimal sample size for each section. By increasing the sample size in each section, the non-conforming item inputs to the assembly line decreases and reduces the expected cost of non-conforming items. Also increasing the sample size increases the inspection time. Therefore, we must choose the optimal sample size by determining the right strategy.

In determining the sample size in each section, the inspection time must be specified. Also, the values of input variables to the assembly line are different in each segment. This problem modeled in the form of nonlinear integer programming problem as follows:

Index:

$$I : \{i | i = 1, 2, \dots, M\}$$

Variables:

n_i : sample size

d_i : The number of non- conforming items in each lot from section i , which varies from zero to N_i .

$$d_i : \{0, 1, 2, \dots, N_i\}$$

Parameters:

N_i : Lot size.

T : Available time of inspection.

C_i : The cost of non- conforming item in the section i if accepted.

C'_i : The cost of a non- conforming item in the section i if rejected.

L : Labor cost per time unit.

r_i : The rate of non- conforming items in the section i .

t_i : Average time of inspection of one item in the section i .

α : Producer risk (probability of rejecting a good lot).

β : Consumer risk (probability of accepting a bad lot).

AQL : Acceptable quality level.

LQL : Limiting quality level.

$b(d_i | N_i, r_i)$: The probability that section i has d_i non- conforming items so that the lot size is N_i and the rate of non- conforming items is r_i .

$h(0 | N_i, d_i, n_i)$: The probability of finding a non- conforming item in the section i such that the lot size is N_i and the number of non- conforming items is d_i and the size of the sample size is n_i .

$$p = \min f(n_1, n_2, \dots, n_M) = \text{Min} \sum_{i=1}^M \left\{ \frac{C_i \sum_{d=0}^{N_i} b(d_i | N_i, r_i) d_i h(0 | N_i, d_i, n_i)}{\sum_{d=0}^{N_i} b(d_i | N_i, r_i) h(0 | N_i, d_i, n_i)} + L t_i n_i \right\} \times \left(\sum_{d=0}^{N_i} b(d_i | N_i, r_i) h(0 | N_i, d_i, n_i) \right) + \left(1 - \sum_{d=0}^{N_i} b(d_i | N_i, r_i) h(0 | N_i, d_i, n_i) \right) \times (N_i t_i L + N_i r_i C_i') \quad (1)$$

$$\sum_{i=1}^M t_i n_i \leq T \quad (2)$$

$$b(d_i | N_i, r_i) = \binom{N_i}{d_i} r_i^{d_i} (1-r_i)^{N_i-d_i} \quad (3)$$

$$h(0 | N_i, d_i, n_i) = \frac{\binom{N_i - d_i}{n_i}}{\binom{N_i}{n_i}} \quad (4)$$

$$\sum_{d=0}^{N_i} b(d_i | N_i, AQL) h(0 | N_i, d_i, n_i) \geq 1 - \alpha \quad (5)$$

$$\sum_{d=0}^{N_i} b(d_i | N_i, LQL) h(0 | N_i, d_i, n_i) \leq \beta \quad (6)$$

$$0 \leq n_i \leq N_i \quad (7)$$

$$n_i = \text{int eger} \quad (8)$$

The goal of the problem is to distinguish the optimal sample size for each section so that the total cost includes the inspection cost, the cost of non- conforming items and the rejecting cost of the lot would be minimum. The equation for total cost can be elaborated as following:

*(Cost of non- conforming items conditioned on accepting the lot + inspection cost) * (probability of acceptance) + (lot rejection probability) * (lot rejection cost + lot total inspection cost)*

The constraint (2) shows the maximum inspection time for inspecting the M parts. For each lot in each part, there are two decisions (acceptance, rejection) and the probability of d_i non- conforming items in the lot of part i follows the binomial distribution as displayed in equation (3). The number of non- conforming items in the sample will be allowed to be zero for accepting the lot and since the inspection is done without replacement of non-conforming items, as a result of the hypergeometric distribution is employed in Equation (4). For evaluating the probability of accepting the lot, constraint (5) denotes the producer risk that evaluates the probability of accepting a good lot. The constraint (6) is the consumer risk and shows the probability of accepting a bad lot. The constraint (7) shows the sample size range from zero to N_i , and the constraint (8) indicates that the sample size values are integer. If the sample size is zero, the inspection will not be done and the risk of accepting a lot with d_i non-conforming items will be high. If the size of the sample is N_i , the inspection is done 100% and the acceptance risk for a lot with d_i non- conforming items is zero, but the inspection cost will be high. By increasing the sample size, the probability of accepting lot with d_i non- conforming items decreases. Therefore, the larger sample size is expected cost to have a less defectives input to the assembly line.

Since we are using zero acceptance number, thus the sample is inspected and if non- conforming items are not found, lot is accepted, hence the acceptance cost includes the inspection cost and cost

of non- conforming items entering the assembly line, so that the cost of a defective input to The assembly line is C_i , thus the cost of accepting non- conforming items entering the assembly line is as following:

$$C_i E(d_i | \text{Accepting the Lot}) = C_i E(d_i | \text{all items are conform in the sample}) = \left(\frac{C_i b(d_i | N_i, r_i) d_i h(0 | N_i, d_i, n_i)}{b(d_i | N_i, r_i) h(0 | N_i, d_i, n_i)} \right)$$

Now, in order to obtain the expected total cost for a given part, taking into account all possible values of d_i , the equation is calculated as following:

$$\left(\frac{C_i \sum_{d_i=0}^{N_i} b(d_i | N_i, r_i) d_i h(0 | N_i, d_i, n_i)}{\sum_{d_i=0}^{N_i} b(d_i | N_i, r_i) h(0 | N_i, d_i, n_i)} \right)$$

If a non- conforming item is found in the sample, then the lot is rejected. In this case, the costs will include the inspecting cost of all items in the lot and the cost of replacing or repairing non- conforming items with the conforming items and it will be calculated as following:

$$(N_i t_i L + N_i r_i C_i')$$

Finally, to find the expected total cost for section M, the values are aggregated and calculated as Equation (1).

It should be noted that the model presented in the reference does not consider the cost of lot rejection, but in the model of this study, in addition to the cost of defective items entering the assembly line and inspection costs, the cost of lot rejection means the replacement cost and defective items replacement and Rectifying inspection is included in the model.

4. Results and analysis:

In this study, we plan to obtain optimal sample size values for 20 different parts ($M = 20$). Inspection time is 8 hours and 10 inspectors are in the process, so the total inspection time is 80 hours ($T = 4800$ min), each inspector's wage rate is \$ 0.05 per minute; as a result, the inspection cost is \$ 240.

Input values are t_i : Inspecting average time for one item. C_i : The cost of non- conforming items entering the assembly line. C_i' : Replacing cost or repairing cost of one non-conforming item. N_i : Lot size. r_i : Rates of non-conforming items.

Because of the added cost of lot rejection and lot Rectifying Inspection, the entire lot will be inspected if a defective item is found and by considering other base article input values, each lot input rate and inspection time are less than the base article values.

4.1. Solution Method

The proposed model is programmed in the MATLAB environment. First the values $d_i = \{0, 1, 2, \dots, N_i\}$ are substituted in the objective function for different parts and the value of cost objective

Function is calculated, then the optimal sample size will be determined by determining the minimum Value of objective function. When the sample size is obtained, the percentage of non- conforming items after the inspection is calculated for different parts. Now, with a non- conforming item price, the inspection cost and the cost of rejecting the lot after the inspection can be calculated. For example, the calculations

Performed for the first part will be as follows:

$$AOQ = \frac{(P_a \times r)(N - n)}{N}$$

In the above relation P_a is the lot accepting probability, and for $N = 168$ and $n = 28$, the AOQ value is 0.0071. The cost of non- conforming items and the lot rejection cost after the inspection can now be obtained as follows:

$$\text{Cost of non- conforming items after inspection} = (N - n) \times C_i \times AOQ = 140 \times 86 \times 0.00713 = 85.8$$

$$\text{Cost of rejecting the lot after inspection} = (N - n) \times C_i' \times AOQ = 140 \times 77 \times 0.00713 = 76.8$$

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Table 4-1 shows the optimal sample size values and the cost of different decisions and reduction percentages in the non-conforming proportion.

Table1. Answer the problem

Number of parts, M	20	α	0.05
Time of inspection available, T	4800	β	0.1
Hourly rate of inspection wage, L (\$/min)	0.05	AQL	0.001
		LQL	0.2

Parts i	Input					Decisions		Effectiveness								
	Inspection Time per item (min) t	NC cost per item (\$), C	Cost of rejection (\$), C'	lot size N	NC rate r	Sample size, n	pct to be inspected n/N	NC rate before inspection n	NC rate after inspection n	pct change in NC rate	NC cost before inspection n	NC cost after inspection n	pct change in NC Cost	Cost of rejection before inspection n	Cost of rejection after inspection n	pct change in Cost of rejection
1	1	86	77.4	168	0.08	28	17	0.08	0.007	91	1156	86	93	1040	77	93
2	0.5	129	116.1	105	0.03	43	41	0.03	0.005	84	406	39	90	366	35	90
3	1.1	121	108.9	145	0.07	27	19	0.07	0.009	88	1228	123	90	1105	111	90
4	0.3	182	163.8	165	0.1	26	16	0.1	0.006	94	3003	158	95	2703	142	95
5	2	60	54	129	0.08	22	17	0.08	0.011	86	619	73	88	557	66	88
6	1	61	54.9	148	0.1	23	16	0.1	0.009	91	903	65	93	813	59	93
7	1.8	40	36	160	0.02	47	29	0.02	0.006	72	128	25	80	115	22	80
8	0.3	76	68.4	100	0.07	15	15	0.07	0.021	70	532	135	75	479	121	75
9	2	111	99.9	110	0.01	51	46	0.01	0.003	68	122	3	97	110	3	97
10	1.6	74	66.6	123	0.09	21	17	0.09	0.011	87	819	86	90	737	77	90
11	1.9	182	163.8	134	0.04	36	27	0.04	0.007	83	976	124	87	878	111	87
12	2.3	189	170.1	170	0.05	35	21	0.05	0.007	86	1607	176	89	1446	159	89
13	2	28	25.2	128	0.07	21	16	0.07	0.013	81	251	40	84	226	36	84
14	0.3	69	62.1	158	0.04	40	25	0.04	0.006	85	436	49	89	392	44	89
15	1.7	67	60.3	170	0.05	33	19	0.05	0.008	85	570	71	88	513	64	88
16	0.8	104	93.6	150	0.04	38	25	0.04	0.007	84	624	77	88	562	69	88
17	2	45	40.5	130	0.06	23	18	0.06	0.012	79	351	60	83	316	54	83
18	0.3	129	116.1	169	0.06	33	20	0.06	0.007	89	1308	117	91	1177	105	91
19	0.2	82	73.8	155	0.1	24	15	0.1	0.008	92	1271	83	93	1144	74	93
20	1.7	49	44.1	150	0.03	40	27	0.03	0.007	78	221	36	84	198	32	84

As shown in Table 1, the first part is the input values that are different for each part, the second part is the results and the problem that shows the optimal sample size, and The third section shows the rate of defective items, the defective items cost, and the lot rejection cost before and after the inspection.

By comparing the model results presented in this article and the base article, the percentage of inspection (N / n) in all sections improved compared to the baseline results. This improvement is also shown in the effectiveness section. The average efficiency of the rate of defective items after inspection in this article is 83.6%, while in the base article it is 46.5%.

Table 2. Total benefits

	Before inspectin \$	After inspection \$	Change \$	Change %
Inspection cost	-	240	240	0
Cost of rejection	14877	1462	13415	90.173
NC cost	16530	1624	14906	90.175
total cost	31407	3326	28561	90.938

As can be seen in Table 2, the quality control costs have dropped significantly after the inspection. Expected total cost is 3326\$, which includes 240\$ for inspection costs, 1462\$ for lot rejection costs and 1624\$ for non- conforming items entering the assembly line.

The following graphs can be used to estimate the number of defective items and the rejection cost and non-conforming items before and after the inspection.

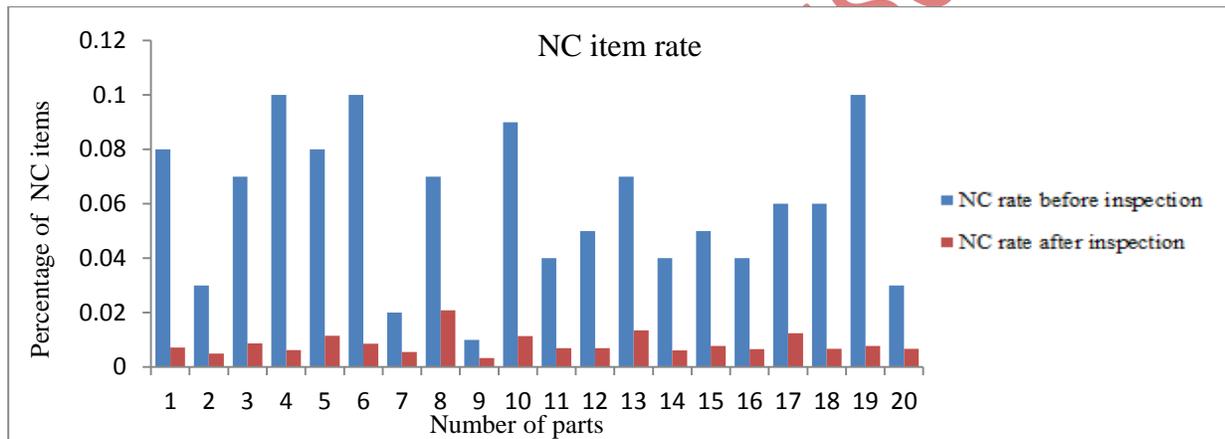


Fig.1. Non-conforming item rates before and after inspection

Fig.1 shows the rate of defective items before and after the inspection as the graph is visible, the rate of defective items decreased significantly after the inspection, indicating that the consideration the cost of lot rejection, i.e. the cost of rejecting and replacing the defective items and rectifying inspection in the model, significantly reduced the inputs to the assembly line. And the quality of the product comes out. Fig.2 and 3 also show defective items cost and the lot rejection cost and show the reduction in costs after inspection.

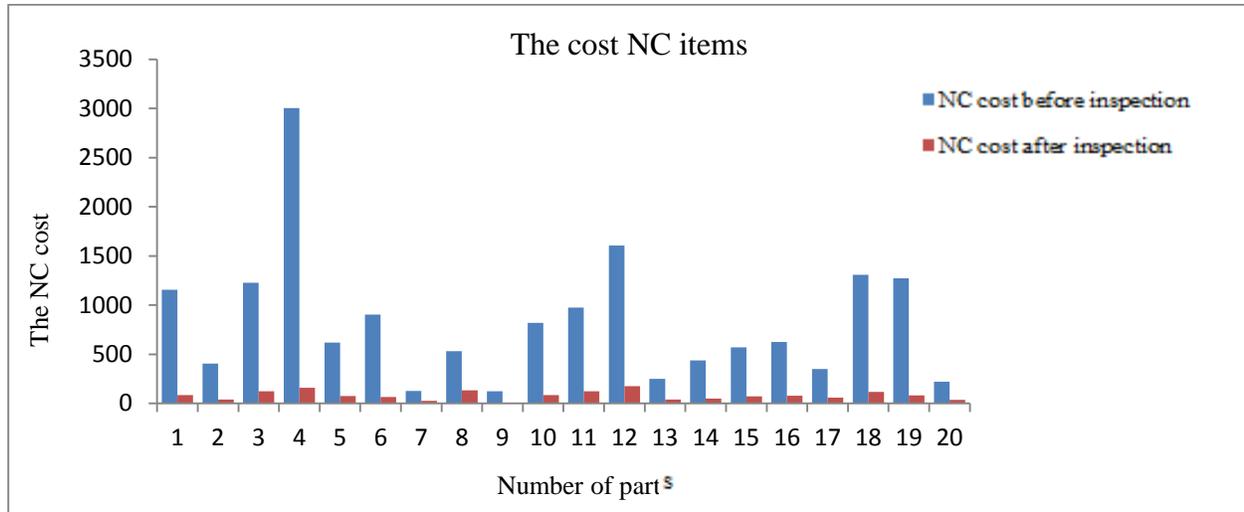


Fig.2. Non-conforming items cost before and after inspection

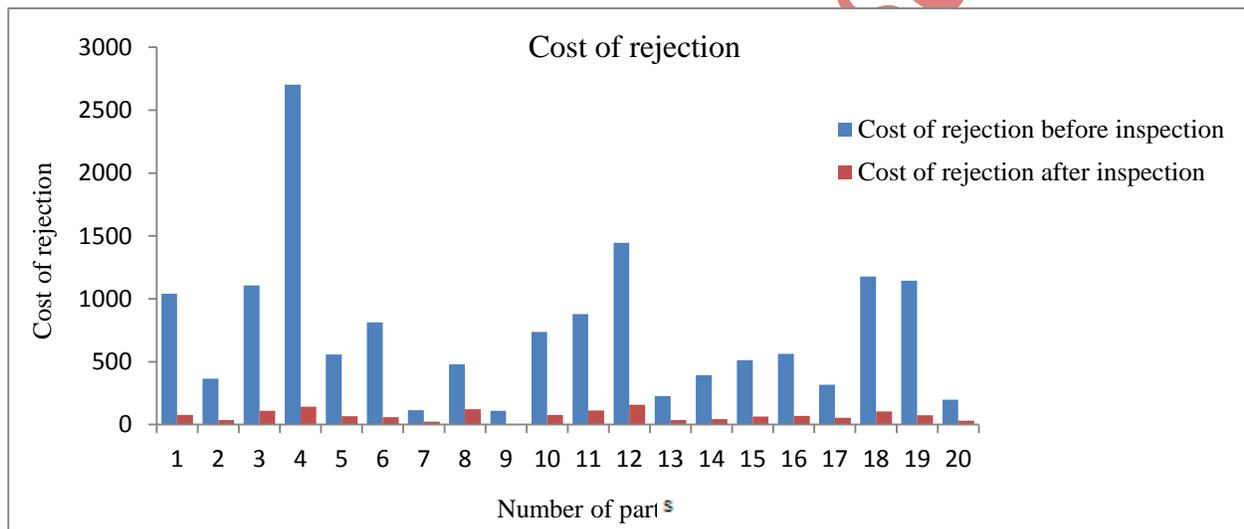


Fig.3. Cost of rejection before and after inspection

By comparing the model presented in this paper and the model presented in Reference [9] and the results of these models, it can be concluded that the cost of lot rejection means the rejection cost and replacing defective items with healthy items and rectifying inspection has greatly reduced costs.

In general, there are three different states for the objective function of the inspection plan in the system:

First state: The objective function for section i is extremely increasing, meaning that the inspecting cost is more than the expected cost of non-conforming items, so inspecting the items is not optimal.

Second state: The objective function is extremely decreasing. It means that the inspecting cost of all items is less than the expected cost of non-conforming items. In this case, the 100% inspection is optimal.

Third state: The objective function is convex. It means, a specific sample size exists that will minimize the total cost of sampling plan. In the third case, the model results in a sampling plan with a given sample size as the solution.

5. Conclusion

The purpose of this research is to find the most economical solution using a mathematical model to distinguish the optimal sample size to control the quality of the products in the input lot. To obtain this goal, the model should determine an optimal balance between the inspection cost, the cost of non-conforming items entering the assembly line and the cost of the lot rejection.

The model of this research is an integer non-linear programming model for designing a single sampling plan with a zero-acceptance number in order to inspect the items entering to the assembly line. The proposed model provides an optimal solution for the problem.

In this research, a practical example with 20 sections is presented to explain the model application. The input parameters were given to the model and the optimal sample size was achieved. In the numerical example, it can be seen that the rate of non-conforming items, the cost of non-conforming items and the lot rejection cost, and the total cost of the quality control, has decreased after implementing the proposed method, and the use of the model by companies and manufacturers reduces costs and thus increases the profits.

The model considered in this study is according to the single sampling plan. In future research, double or other sampling schemes could be used to compare with a single sampling plan. Also, in the proposed model, the acceptance number is zero; in future models, other acceptance numbers can be analyzed. In addition, it is possible to consider the parameters as fuzzy instead of constant and compare the results with the results. In the proposed model the objective is reducing the quality control cost, thus a model with the objective of reducing the average number of inspections can be designed.

References

- [1] Montgomery, Douglas, C. “*Statistical Quality Control*”, Noorossana, R., Iran University of Science and Technology, (2013), (In Persian)
- [2] Wetherill, G.B., and Chiu, W.K. “A review of acceptance sampling schemes with emphasis on the economic aspect”, *International Statistical Review*, 43, pp. 191-210, (1975).
- [3] Hald, A. “*Statistical theory of sampling inspection by attributes*”, Academic Press, New York, U.S.A, (1981).
- [4] Lieberman, G. I. and Resnikov, G.J. “Sampling plans for inspection by variables”, *Journal of the American Statistical Associative*, 50, pp. 457-516 (1955).
- [5] Bennett, G.K.; Schmidt, J.W.; Case, K.E. “The choice of variables sampling plans using cost effective criteria”, *AIIE Transactions*, 6, pp. 178-184 (1974).

- [6] Schmidt, J.W.; Bennet, G.K.; Case, K.E. "Three action cost model for acceptance sampling by variables", *Journal of Quality Technology*, 16(3), pp. 10-18 (1980).
- [7] Taguchi, G. "Quality evaluation for quality assurance", American supplier institute, Romulus, Michigan, U.S.A, (1984).
- [8] Zlatan Hamzic, Elizabet A. Cudney, and Ruwen Qin. "Development of an optimization model to determine sampling levels", Missouri University of science and technology (2013).
- [9] Ruwen Qin, Elizabet A. Cudney, Zlatan Hamzic. "An optimal plan of zero-defect single-sampling by attributes for incoming inspections in assembly lines", *European Journal of Operational Research* 246 (2015) 907–915, (2015).
- [10] Willemain, T. R. "Estimating the population median by nomination sampling", *Journal of the American Statistical Association*, 75(372), 908-911, (1980).
- [11] Ferrell W.G. and Choker jr.A. "Design of economically optimal acceptance sampling plane with inspection error", *Computer & Operations Research*, Vol.29, pp. 1283-1300, (2002).
- [12] Niaki, S. A., Nezhad, M. F. "Designing an optimum acceptance sampling plan using Bayesian inferences and a stochastic dynamic programming approach", *Scientia Iranica Transaction E-Industrial Engineering*, 16(1), 19-25, (2009).
- [13] Nezhad, M. S. F., Nasab, H. H. "Designing a single stage acceptance sampling plan based on the control threshold policy. *International Journal of Industrial Engineering*", 22(3), 143-150, (2011).
- [14] Hsu, L. F., Hsu, J. T. "Economic design of acceptance sampling plans in a two-stage supply chain", *Advances in Decision Sciences*, (2012).
- [15] Fallah Nezhad, M. S., Akhavan Niaki, S. T. "A new acceptance sampling policy based on number of successive conforming items", *Communications in Statistics-Theory and Methods*, 42(8), 1542-1552, (2013).
- [16] Fallahnezhad MS, Aslam M. "A New Economical Design of Acceptance Sampling Models Using Bayesian Inference", *Accreditation & Quality Assurance*, Vol. 18, pp. 187-195, (2013).
- [17] Li, M. H. C., Al-Refaie, A., Tsao, C. W. "A Study on the Attributes Sampling Plans in MIL-STD-1916", *Lecture Notes in Engineering and Computer Science*, 2190, (2011).
- [18] Champernowne, D. G. "The economics of sequential sampling procedures for defectives", *Applied Statistics*, 118-130, (1953).
- [19] Barnard, George A. "Sampling inspection and statistical decisions", *Journal of the Royal Statistical Society. Series B (Methodological)*, 151-174, (1954).
- [20] Hamaker, H. C. "Some basic principles of sampling inspection by attributes", *Applied Statistics*, 149-159, (1958).
- [21] Calvin, T. "Quality Control Techniques for Components, Hybrids, and Manufacturing Technology", *IEEE Transactions on components, hybrids and manufacturing technology*, 6(3), 323-328, (1983).

- [22] Salameh, M. K., and Jaber, M. Y. "Economic production quantity model for items with imperfect quality", *International journal of production economics*, 64(1), 59-64, (2000).
- [23] Maddah, B., Jaber, M. Y. "Economic order quantity for items with imperfect quality", Revisited. *International Journal of Production Economics*, 112(2), 808-815, (2008).
- [24] Taghipour, S., and Banjevic, D. "Periodic inspection optimization models for a repairable system subject to hidden failures", *Reliability, IEEE Transactions on*, 60(1), 275-285, (2011).
- [25] Taghipour, S., Banjevic, D., and Jardine, A. K. "Periodic inspection optimization model for a complex repairable system", *Reliability Engineering & System Safety*, 95(9), 944-952, (2010).
- [26] Shi, J., and Zhou, S. "Quality control and improvement for multistage systems: A survey", *IIE Transactions*, 41(9), 744-753, (2009).
- [27] Starbird, S.A. "Acceptance sampling, imperfect production, and the optimality of zero defects", *Naval Research Logistics (NRL)*, 44(6), 515-530, (1997).
- [28] Dodge, H.F., Romig, H.G. "Sampling inspection tables: single and double sampling", (2nd ed.). New York: John Wiley, (1998).
- [29] Arturo J. Fernández. "Economic lot sampling inspection from defect counts with minimum conditional value-at-risk", *European Journal of Operational Research* 258, 573-580, (2017).
- [30] Arturo J. Fernández. "Optimal attribute sampling plans in closed-forms", *Computers & Industrial Engineering* 137, 106066, (2019).
- [31] Lu Cui, Lanju Zhang, Bo Yang. "Optimal adaptive group sequential design with flexible timing of sample size Determination", *Contemporary Clinical Trials* 63, 8-12, (2017).
- [32] Andreas Sommer, Ansgar Steland. "Multistage acceptance sampling under nonparametric dependent sampling designs", *Journal of Statistical Planning and Inference* 199, 89-113, (2019).
- [33] Jafar Ahmadi, Elham Basiri, S.M.T.K. MirMostafae. "Optimal random sample size based on Bayesian prediction of exponential lifetime and application to real data", *Journal of the Korean Statistical Society* 45, 221-237, (2016).