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RESEARCH PAPER

Evaluating Parameter Estimation Effect on the Polynomial Profile Monitoring Methods' Phase II Performance

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Abstract

In several statistical process monitoring applications, it is possible to determine the quality of a product or process using a linear or nonlinear regression relationship called "profile". Basically, standard monitoring methods involve two phases: Phase I and II. Usually, there is a general assumption about knowing the process parameters; yet, this condition is not met in several applications, and parameter estimation takes place using the in-control data set gathered in Phase I. The present study evaluates and compares some Phase II control chart approaches to monitor the second-order polynomial profiles when the parameters of the process are estimated. These methods include Orthogonal, MEWMA and dEWMA-OR control charts. Each control chart performance is measured concerning ARL, SDRL, AARL and SDARL metrics using the Monte Carlo simulation approach. The results showed that parameter estimation strongly affects the in-control and out-of-control performance of control charts, particularly in the case of using only a few Phase-I samples for the parameter estimation. Moreover, the superior overall performance of the Orthogonal method rather than the other competing methods is shown. Furthermore, we concluded that the F estimation method leads to better control chart performance in Phase II.

Keywords:
Profile Monitoring;
Polynomial Profile;
Estimation Effect;
Control Chart;
Run Length;
Statistical Process Control

Introduction

The profile stability during the time is monitored using control charts. Profiles are classified based on the structure of the association of the explanatory and the response variables, which are monitored by different methods in Phase I and Phase II [1].

Phase I is a retrospective analysis that aims at determining the process stability and parameter estimation. Nevertheless, Phase II analysis aims at prompt detection of the changes in the process parameters. The use of different metrics aims to evaluate and compare control charts in Phases I and II. In Phase I, a commonly used measure is the false alarm rate, indicating the probability of occurrence of at least one false alarm, whereas in Phase II some properties of the run length distribution are used for performance evaluation.

For monitoring simple linear profile, which is a simple linear regression model between a response and an explanatory variable, Kang and Albin [2], Kim et al. [3], Stover and Brill [4], Mahmoud and Woodall [5], Mahmoud et al. [6], Mahmoud et al. [7] and Noor al-Sana et al. [8]

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proposed some methods in Phase I or II. When there is more than one explanatory variable in a linear regression model, we are dealing with a multiple linear profile, which has been investigated by Mahmoud [9], Jensen et al. [10], Amiri et al. [11], Parker and Finley [12]. Moreover, Noorossana et al. [13], Eyvazian et al. [14], Ayoubi et al. [15], Zou et al. [16] suggested several ways to monitor multivariate linear profiles, including a linear regression relationship of multiple response variables with one or more explanatory variables. Nonlinear profiles, which are more complicated than other types of profiles due to the nonlinear regression relationship of response and explanatory variables, are also studied by Ding et al. [17], Williams et al. [18], Vaghefi et al. [19] and Williams et al. [20] by two parametric and nonparametric methods.

Many investigations of Phase II profile monitoring are based on the presumption of knowing the in-control parameters and designing the control chart according to known parameters. This assumption simplifies the development and evaluation of the control chart, but in practical environments, there are unknown parameters of the process, requiring estimations from in-control phase I samples. The estimator variability affects the chart performance when the estimated parameters are used rather than known parameters to design a control chart. Woodall and Montgomery [21] explicitly suggested that more work is required to examine the effects of estimating parameters on the performance of control charts. Many researchers have investigated the effects of the estimated parameters on the control chart performance, including Chen [22], Chakraborti [23], Albers and Kallenberg [24] Jones et al. [25,26], Maravelakis and Castagliola [27], Mahmoud and Maravelakis [28], Shu et al. [29], Castagliola and Maravelakis [30], Champ et al. [31], Castagliola et al. [32].

In Phase I, the estimation technique with the least effects on the control chart performance in Phase II is selected as the more appropriate method. Therefore, in order to evaluate this effect, a reasonable metric is needed. Run length (RL) represents the number of samples considered to achieve an out-of-control signal by the chart. Many related works have used the average run length (ARL) metrics to measure the parameter estimation effects on Phase II monitoring. However, given the skewness of run length distribution with the assumption of unknown process parameters, applying ARL along with standard deviation of run length (SDRL) leads to a better judgment of Phase II performance. On the other hand, Jones and Steiner [33] and Zhang et al. [34,35] used the standard deviation of ARL (SDARL) as a new performance metric. Taking independent phase I samples, various estimation values and consequently different in-control ARL are obtained. Thus, a novel variability source, namely practitioner to practitioner variability, is added to the process that causes ARL to become a random variable. Therefore, the chart performance is evaluated using the mean and standard deviation of ARL. Researchers usually set the average of ARL (AARL) value equal to the designated in-control value of ARL and SDARL between 5 to 10 percent of the desired in-control ARL [36]. Obviously, in the case of knowing the process parameters, the SDARL value will be expectedly equal to zero.

So far, the measurement of the parameter estimation effects on Phase II control chart performance in profile monitoring has been considered in a few studies. Woodall and Montgomery [37] proposed this subject as an issue that needs more attention. Mahmoud [38] considered methods suggested in the studies of Kang and Albin [2], Kim et al. [3] and Mahmoud et al. [7] to monitor simple linear profiles. These three methods were evaluated and compared using estimated parameters concerning ARL and SDRL metrics. According to the simulation results, the estimated parameters strongly affect the mentioned methods' performance in Phase II. They also showed that a higher number of Phase I samples results in better in-control ARL, which should nearly equal the desired in-control ARL. However, in most of the practical examples, taking many samples required a great amount of cost and time. Therefore, to overcome this challenge, they used corrected limits wider than the original control limits according to known parameters. Applying corrected limits allows practitioners to take fewer samples from the process when the parameters are unknown. The results of simulations proposed in the study of Kang and Albin [2] has a better in-control performance than other rival techniques, and the method proposed in the study of Mahmoud et al. [7] shows the best overall out-of-control performance. Aly et al. [36] selected three monitoring methods, similar to Mahmoud [38], and compared them based on the SDARL metric, which includes practitioner-to-practitioner variability. As they indicated, the method of Kim et al. [3] had the smallest in-control SDARL value, which shows the superior performance of this method rather than the other ones. Yazdi et al. [39] evaluated the impacts of the estimated parameters on the multivariate simple linear profiles. They compared three control chart approaches, including MEWMA, MEWMA-3 and MEWMA / χ^2 introduced by Noorossana et al. [13] concerning AARL, SDARL and CVARL metrics. The authors claimed that the CVARL metric, which was previously applied by Aly et al. [40] to investigate the performance of multivariate adaptive EWMA control chart, leads to more reliable decisions. According to the results, the estimated parameters significantly influence the in-control and out-of-control performance. When the process is out-of-control, they showed that MEWMA and MEWMA / χ^2 techniques have a better performance compared to MEWMA-3 concerning CVARL.

The polynomial profiles, on which the present work primarily concentrates, describe a polynomial regression model between explanatory and response variables. Kazemzadeh et al. [41] introduced three methods to monitor polynomial profiles in Phase I: the change point approach, the F-approach as well as the T^2 method. Zou et al. [42] introduced a new MEWMA control chart to monitor general linear profiles, which can also be applied for polynomial profiles monitoring in Phase II. Kazemzadeh et al.[43] investigated polynomial profiles Phase II monitoring using a new technique based on orthogonal transformation. Amiri et al. [44] provided a case study in the automotive industry. They considered a second order polynomial regression between the torque that an engine produced and the engine speed in revolutions/min when autocorrelation was present within each profile (sample). For checking the process stability in Phase I, they used a T^2-based procedure after reducing the number of process parameters. They monitored Phase II using a linear mixed model method whose development was carried out by Jensen et al. [10]. Kazemzadeh et al. [45] considered polynomial profiles when autocorrelations were present between samples, suggesting two methods to monitor autocorrelated polynomial profiles using a first order autoregressive model (AR(1)) and according to time series analysis approaches. Abdella et al. [46] developed two approaches using a double exponentially weighted moving average (dEWMA) control chart for polynomial profiles Phase II monitoring, and simulations showed a desirable performance of both techniques.

As far as the authors know, the impact of estimated parameters on the control charts performance in Phase II monitoring of polynomial profile is a research gap that has not been investigated, yet. In this study, two estimation methods in Phase I monitoring of polynomial profiles consisting of F-approach and T^2 approach provided by Kazemzadeh et al. [41], are selected and the impact of estimated parameters on the performance of three Phase II methods is evaluated and compared. These methods are the MEWMA control chart that Zoe et al. introduced [42], the Orthogonal method that Kazemzadeh et al. introduced [43] and the dEWMA-OR chart proposed by Abdella et al. [46]. Selecting two estimation methods (F-Approach and T^2 Approach) is due to their superior performance than the other one.

The main contribution of this research is summarized as follows for clarification purposes:

- Evaluating the impact of estimating parameters in Phase I on the performance of control charts in Phase II for polynomial profiles, that has not been investigated for this type of profile so far.
- Using AARL and SDARL metrics with better performance evaluation for evaluating the in-control performance

- Investigate the effect of sample size for estimating parameters in phase I on Phase II methods performance
- Determining a proper method for estimating parameters in phase I of polynomial profile that leads to better performance of Phase II control charts
- Determining proper methods to monitor phase II polynomial profile in the case of parameter estimation in phase I

The present article has the following structure: Section 2, presents some control charts to monitor polynomial profiles in Phase I and II. Section 3 evaluates the effect of estimated parameters on the performance of selected methods. Finally, Section 4 summarizes conclusions, along with suggestions concerning further research.

Control charts to monitor polynomial profiles in Phase I and II

A polynomial profile of p order with only one explanatory variable is defined as follows:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \cdots + \beta_p X^p + \varepsilon \tag{1}$$

For each sample, the i^{th} observation is determined in the form of (x_{ij}, y_{ij}) where y_{ij} follows the following relationship with explanatory variable:

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + \beta_2 X_{ij}^2 + \cdots + \beta_p X_{ij}^p + \varepsilon_{ij} \qquad i = 1,2,\dots,n_j \qquad j = 1,2,\dots \tag{2}$$

In the above model, ε_{ij} represents a random variable, following $N\sim(0, \sigma^2)$ normal distribution. In most of the related studies, fixed values are assumed for the explanatory variable. Therefore, Eq. 2 can be rewritten such as:

$$Y_{ij} = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \cdots + \beta_p X_i^p + \varepsilon_{ij} \qquad i = 1,2,\dots,n \qquad j = 1,2,\dots \tag{3}$$

Phase I control charts

F-Approach

Kazemzadeh et al. [41] proposed this control chart for polynomial profiles Phase I monitoring and is an extension of the F method that Mahmoud and Woodall [5] suggested to monitor simple linear profiles. This method merges all m samples of size n_j into a big sample with a size of $N = \sum_{j=1}^{m} n_j$, then the following relation defines $m-1$ indicator variable:

$$Z_{ji} = \begin{cases} 1 & \text{if observation } i \text{ belongs to sample } j \\ 0 & \text{otherwise} \end{cases} \qquad i = 1,2,\dots,N \qquad j = 1,2,..,m-1 \tag{4}$$

Fitting of the merged data to the following model takes place:

$$Y_i = \beta_0 + \beta_1 X_i + \cdots + \beta_p X_i^p + \beta_{01} Z_{1i} + \beta_{11} Z_{1i} X_i + \cdots + \beta_{p1} Z_{1i} X_i^p + \cdots + \beta_{0m'} Z_{m'i} + \beta_{1m'} Z_{m'i} X_i + \cdots + \beta_{pm'} Z_{m'i} X_i^p + \varepsilon_i \qquad i = 1,2,\dots,N \tag{5}$$

where $m' = m - 1$ and m^{th} sample is known as the reference sample.
In order to estimate the parameters of this model, a polynomial regression model in Eq. 6 is used with a separate fitting of every sample m times.

$$Y_{ij} = \beta_{0j} + \beta_{1j} x_{ij} + \beta_{2j} x_{ij}^2 + \cdots + \beta_{pj} x_{ij}^p + \varepsilon_{ij} \quad i = 1,2,\dots,n_j \quad j = 1,\dots,m \tag{6}$$

The following hypothesis is tested to examine all m models equality:

$$\begin{cases} H_0: & \beta_{01} = \beta_{11} = \cdots = \beta_{p1} = \cdots = \beta_{0m'} = \beta_{1m'} = \cdots = \beta_{pm'} = 0 \\ H_1: & \text{H0 is not true} \end{cases} \tag{7}$$

The null hypothesis leads to the following reduced model:

$$Y_i = \beta_0 + \beta_1 X_i + \cdots + \beta_p X_i^p + \varepsilon_i \qquad i = 1,2,\ldots,N \tag{8}$$

The following relation shows the standard test statistic for H_0 testing:

$$F = \frac{SSE(R) - SSE(F)/(df_R - df_F)}{SSE(F)/df_F} \tag{9}$$

In which SSE(F) and SSE(R) represent the residual sum of squares obtained through model fitting (5) and (8). The test statistic follows F distribution with $(p+1)(m-1)$ and $N-(p+1)m$ degrees of freedom subject to the null hypothesis.

Mahmoud and Woodall [5] suggested a univariate control chart for error variance monitoring in conjunction with the global F-test concerning linear profiles, when $n_j = n$. Kazemzadeh et al. [41] extended it for variable sample sizes for polynomial profiles. In their study, the test statistic is F_j, shown in Eq. 10, which follows F distribution with $(n_j - p - 1)$ and $\sum_{i\neq j}^{m}(n_i - p - 1)$ degrees of freedom subject to the null hypothesis.

$$F_j = \frac{((n_j-p-1)(MSE_j)/\sigma^2(n_j-p-1))}{(\sum_{i\neq j}^{m}(n_i-p-1)\, MSE_i/\sigma^2 \sum_{i\neq j}^{m}(n_i-p-1))} \tag{10}$$

The lower and upper control limits regarding the F statistic is obtained by:

$$UCL = F_{n_j-p-1, \sum_{i\neq j}^{m}(n_i-p-1), \alpha/2} \qquad LCL = F_{n_j-p-1, \sum_{i\neq j}^{m}(n_i-p-1), 1-\alpha/2} \tag{11}$$

T^2 Approach

Kazemzadeh et al. [41] introduced another Phase I method to monitor polynomial profiles, known as T^2 control chart and using the following relation to obtain the chart statistic:

$$T_j^2 = (\hat{\beta}_j - \bar{\bar{\beta}})^T\, \hat{\Sigma}^{-1}\, (\hat{\beta}_j - \bar{\bar{\beta}}) \tag{12}$$

where $\hat{\beta}_j$ is the estimation of profile parameters for sample j, which is obtained using the least squares method as $= (X'X)^{-1}X'Y$ where X and Y represent the observation matrix and the response vector, respectively. $\bar{\bar{\beta}}$ is the average vector of $\hat{\beta}_j$s over m samples as $\bar{\bar{\beta}} = \frac{1}{m}\sum_{j=1}^{m}\hat{\beta}_j$.

Estimation of the covariance matrix takes place according to successive differences in parameter estimators. Williams et al. [18] described the vector $\hat{v}_i$ as $\hat{v}_i = \hat{\beta}_{i+1} - \hat{\beta}_i$, $i = 1,2,\ldots,m-1$. Therefore, $\hat{V}$ is a $(m-1) * (p+1)$ matrix which is obtained according to Eq. 13.

$$\hat{V} = \begin{bmatrix} \hat{v}_1^T \\ \hat{v}_2^T \\ \vdots \\ \hat{v}_{m-1}^T \end{bmatrix} \tag{13}$$

They estimated the variance-covariance matrix by $S = \frac{\hat{V}^T \times \hat{V}}{2(m-1)}$ and calculated the chart statistic as follows:

$$T_j^2 = (\hat{\beta}_j - \bar{\bar{\beta}})^T \, S^{-1} \, (\hat{\beta}_j - \bar{\bar{\beta}}) \tag{14}$$

A specified value of Type I error probability is obtained by selecting the upper control limit for this control chart.

Phase II

MEWMA method

To simultaneously monitor all parameters of general linear profiles, such as polynomial profiles, in Phase II, Zoe et al. [42] suggested using a multivariate exponentially weighted moving average (MEWMA) control chart, introduced firstly in the study of Lori et al. [47]. In this method, for each sample j, $Z_j(\beta)$ and $Z_j(\sigma)$ are calculated according to Eqs. 15 and 16.

$$Z_j(\beta) = (\hat{\beta}_j - \beta)/\sigma \tag{15}$$

$$Z_j(\sigma) = \emptyset^{-1}\{F(\frac{(n-p-1)\hat{\sigma}_j^2}{\sigma^2} \, ; n - p - 1)\} \tag{16}$$

where $\hat{\beta}_j = (X'X)^{-1}X'y_j$, $\hat{\sigma}_j^2 = \frac{1}{n-p-1}(Y_j - X\hat{\beta}_j)^T \, (Y_j - X\hat{\beta}_j)$, $\emptyset^{-1}(.)$ represents the standard normal cumulative distribution function inverse and $F(.;\, \nu)$ represents the chi-squared distribution function having ν freedom degrees.

Z_j vector is determined as $(\, Z_j'(\beta), Z_j(\sigma))'$, which is a $(p+2)$-variate random vector and has a multivariate normal distribution with zero mean vector and covariance matrix of $\Sigma = \begin{bmatrix} (X'X)^{-1} & 0 \\ 0 & 1 \end{bmatrix}$ for an in-control process. Accordingly, W_j statistic is calculated for each sample as follows:

$$W_j = \lambda Z_j + (1-\lambda)W_{j-1} \qquad j = 1,2, \ldots \tag{17}$$

where W_0 represents a $(p+2)$-dimensional starting vector and λ indicates the smoothing constant $(0 < \lambda \leq 1)$. Finally, U_j statistic is calculated as follow:

$$U_j = W'_j \, \Sigma^{-1} \, W_j \tag{18}$$

The chart signals when $U_j > L\frac{\lambda}{2-\lambda}$, where selection of $L > 0$ aims at achieving a certain in-control ARL.

Orthogonal technique

In this method that Kazemzadeh et al. [43] introduced, the transformation of polynomial regression into orthogonal polynomial regression takes place. So, there are independent regression parameters and they can be monitored, independently. However, applying this technique can be criticized in the case of studying a high-order polynomial profile (for example when $p > 2$) because of too many charts required to monitor the process. In the case of an in-control process, the transformed regression models in Phase II can be written in the following form:

$$y_{ij} = B_0P_0(x_i) + B_1P_1(x_i) + B_2P_2(x_i) + \cdots + B_pP_p(x_i) + \varepsilon_{ij} \tag{19}$$

where $P_u(x_i)$ shows a u^{th} order orthogonal polynomial described as follow:

$$\begin{array}{lll} \sum_{i=1}^{n} P_r(x_i)P_s(x_i) = 0 & r \neq s & r,s = 0,1,\dots,p \\ P_0(x_i) = 1 & i = 1,2,\dots,n & \end{array} \tag{20}$$

The least squares estimator of B_{lj} is:

$$\hat{B}_{lj} = \frac{\sum_{i=1}^{n} P_l(x_i)y_{ij}}{\sum_{i=1}^{n} P_l^2(x_i)} \qquad i = 1,2,\dots,n \qquad \mathrm{j} = 1,2,\dots \qquad \mathrm{l} = 0,1,\dots,\mathrm{p} \tag{21}$$

Using separate EWMA charts for monitoring profile parameters, chart statistic is calculated by:

$$EWMA_l(j) = \lambda\hat{B}_{lj} + (1-\lambda)EWMA_l(j-1) \qquad j = 1,2,\dots \tag{22}$$

where λ is smoothing constant and $EWMA_l(0) = B_l$
The following relation shows the upper and lower control limits for the chart statistic:

$$\begin{aligned} LCL &= B_l - K_l\sqrt{\frac{\lambda}{(2-\lambda)} \cdot \frac{\sigma^2}{\sum_{i=1}^{n} P_l^2(x_i)}} \\ UCL &= B_l + K_l\sqrt{\frac{\lambda}{(2-\lambda)} \cdot \frac{\sigma^2}{\sum_{i=1}^{n} P_l^2(x_i)}} \end{aligned} \tag{23}$$

In which, the selection of $K_l(> 0)$ aims at achieving a certain in-control ARL.

Based on the approach that Crowder and Hamilton [48] introduced, the EWMA statistic, and the following relations are used to obtain the upper control limit to monitor the error variance:

$$EWMA_E(j) = \max\{\lambda(MSE_j - 1) + (1-\lambda)EWMA_E(j-1), 0\} \qquad j = 1,2,\dots \tag{24}$$

$$UCL = L_E\sqrt{\frac{\lambda\, Var(MSE_j)}{2-\lambda}} \tag{25}$$

where $EWMA_E(0) = 0$, $MSE_j = (\sum_{i=1}^{n} e_{ij}^2)/n$, $Var(MSE_j) = \frac{2\sigma^4}{n}$ and the selection of L_E (>0) aims at achieving a certain in-control ARL.
For a second-order polynomial regression and the case of equally-spaced levels of x, Kazemzadeh et al. [43] obtained the regression parameters of the transformed model according to the regression parameters of the original model by:

$$\begin{cases} B_2 = \frac{\beta_2 d^2}{\lambda_2} \\ B_1 = \frac{d}{\lambda_1}(\beta_1 + 2\beta_2\bar{x}) \\ B_0 = \beta_0 + \bar{x}\beta_1 + \left[\bar{x}^2 + (\frac{n^2-1}{12})d^2\right]\beta_2 \end{cases} \tag{26}$$

Where regression method in the original model is least squares and d denotes the distance of the x levels, and $\lambda_j s$ have constants values. This method develops the step shifts occurring in the transformed model parameters and results in faster detection in Phase II.

dEWMA-OR method

Basically, Phase II profile monitoring methods performance according to EWMA statistic in detecting small process shifts is remarkable. Abdella et al. [46] used the double EWMA (dEWMA) statistic that Shamma et al. [49] proposed, with less variability and greater smoothing properties compared to the EWMA chart [49]. In the dEWMA-OR method, the orthogonal model is used instead of the original polynomial profile, and each parameter is monitored by a separate dEWMA chart. This technique is a modified version of the Orthogonal method that Kazemzadeh et al. [43] introduced. The dEWMA chart statistics for monitoring b_l in j^{th} sample is as follows.

$$dEWMA_{b_l(j)} = \lambda_2^{(b_l)}\, EWMA_{b_l(j)} + \left(1 - \lambda_2^{(b_l)}\right) dEWMA_{b_l(j-1)} \qquad l = 0,1,\dots,p$$

where

$$EWMA_{b_l(j)} = \lambda_1^{(b_l)}\, b_{lj} + \left(1 - \lambda_1^{(b_l)}\right) EWMA_{b_l(j-1)} \qquad j = 1,2,\dots \tag{27}$$

where $\lambda_1^{(b_l)}, \lambda_2^{(b_l)} \in (0,1)$ are the smoothing constants and $dEWMA_{b_l(0)} = EWMA_{b_l(0)} = B_l$. Calculation of the upper and lower control limits for the dEWMA is as follows:

$$LCL = B_l - L_l\sqrt{\sigma^2_{dEWMA(b_l)}}$$
$$UCL = B_l + L_l\sqrt{\sigma^2_{dEWMA(b_l)}} \tag{28}$$

where $L_l > 0$ is the sigma level and $\sigma^2_{dEWMA(b_l)}$ represents the $dEWMA_{b_l}$ statistic variance. In a polynomial profile where $\lambda_1^{(b_l)} = \lambda_2^{(b_l)} = \lambda$, the following shows estimation of the asymptotic $dEWMA_{b_l}$ statistic variance:

$$\sigma^2_{dEWMA(b_l)} = \sigma^2_{(b_l)} \frac{\lambda(2-2\lambda+\lambda^2)}{(2-\lambda)^3} \tag{29}$$

where the variance of the least square estimator of b_l ($\sigma^2_{(b_l)}$) is obtained by:

$$\sigma^2_{(b_l)} = \frac{\sigma^2}{\sum_{i=1}^{n} P_l^2(x_i)} \tag{30}$$

For monitoring error variance, Abdella et al. [46] proposed another univariate dEWMA chart with following statistics:

$$dEWMA_{E(j)} = Max\{\lambda_{2E}\, EWMA_{E(j)} + (1-\lambda_{2E})dEWMA_{E(j-1)}\,, 0\}$$
$$EWMA_{E(j)} = \lambda_{1E}\left(MSE_j - 1\right) + (1-\lambda_{1E})EWMA_{E(j-1)} \tag{31}$$

λ_{1E} and λ_{2E} are smoothing parameters that are always considered as constant values.

The method obtains the upper control limit as follows:

$$UCL_E = L_E\sqrt{\sigma^2_{dEWMA(E)}} \tag{32}$$

where $L_E > 0$ is the sigma level, and $\sigma^2_{dEWMA(E)}$ represents the MSE_j variance. When $\lambda_{1E} = \lambda_{2E} = \lambda_E$, the following equation shows the estimation of the MSE_j asymptotic variance:

$$\sigma^2_{dEWMA(E)} = \frac{\lambda_E(2-2\lambda_E+\lambda_E^2)}{(2-\lambda_E)^3}\, \sigma^2_{MSE} = \frac{2\lambda_E\sigma^4(2-2\lambda_E+\lambda_E^2)}{n(2-\lambda_E)^3} \tag{33}$$

The main problem of the dEWMA-OR method, like the Orthogonal method, is the large number of control charts when a large order polynomial profile is under study.

The estimated parameters' impacts on the performance of phase II polynomial profile approaches

To evaluate the estimated parameters' effects on Phase II performance, a second order in-control polynomial profile model is considered as Eq. 34. Montgomery et al. [50] don't recommend using a higher order polynomial profile, unless with justifying reasons or excuses.

$$y_{ij} = 3 + 2x_i + x_i^2 + \varepsilon_{ij} \qquad i = 1,2,\ldots,10 \qquad j = 1,2,\ldots \tag{34}$$

According to the above model, $\beta_0 = 3$, $\beta_1 = 2$ and $\beta_2 = 1$.

y_{ij} denotes the value of the response variable in i^{th} observation of j^{th} sample. ε_{ij} is independent error variable with a distribution of $N \sim (0,1)$. The values of the explanatory variable (x_i) are assumed to be fixed in all samples as $x = 1, 2, 3, \ldots, 10$.

Note that for decreasing the effect of multicollinearity in MEWMA method, subtraction of average x-values takes place from every x. So, x values of -4.5, -3.5, -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, 3.5, 4.5 can be obtained.

The smoothing parameter (λ) for all methods is considered as 0.2. This value is usually applied in many related researches due to proper performance in detecting small shifts in Phase II such as [42], [43], and [46].

The in-control performance

In this study, different values of sample size (m) such as 30, 100, 500, 1000, 2000 and 5000 are selected for Phase I estimation. Also, different metrics, including ARL, SDRL, AARL and SDARL are calculated using the Monte Carlo simulation method for the evaluation of parameter estimation effects on the performance of Phase II. The following presents the steps of the suggested Monte Carlo simulation algorithm:

1. Control limits of underlying control charts are chosen so that produce an overall in-control $ARL = 200$ with the assumption of knowing the parameters. Kazemzadeh et al. [43] and Abdella et al. [46] introduced these limits which are shown in Table 1.
2. m profiles are generated based on known parameters using $N \sim (0,1)$ and the estimation of profile parameters is carried out using the F or T^2 method.
3. A random sample with $n = 10$ is generated indicating a new sample in Phase II.
4. Calculation of the chart statistic is according to the estimated parameters and comparisons with the control limits as Table 1 shows.
5. Repetition of steps 3-4 continues until sending an out-of-control signal by the chart, and run length (RL) is recorded.
6. Repetition of steps 3-5 continues 10,000 times, obtaining in-control ARL and SDRL values.

Parameter estimation in Phase 1 was performed using different sample sizes (m) such as 30, 100, 500, 1000, 2000 and 5000, and the results were compared to examine the sample size effects on the performance of Phase II monitoring approaches. Note that, the simulation replication is set as 10000. A similar approach has been applied by Kazemzadeh et al. [44], Mahmoud [38], Aly et al. [36], Yazdi et al. [39].

In addition to the ARL and SDRL metrics, the use of the average and ARL (AARL and SDARL) standard deviation aims at evaluating the parameter estimation effects, which reflects the practitioner-to-practitioner variations, as Zhang et al. [34] have argued. For this aim, in Step

2, Phase I estimates are generated and repetition of steps 2-6 continues 10,000 times. Then, the AARL and SDARL values are calculated.

Table 1. Control limits of three competing methods under in-control condition

Method	Parameter	LCL	UCL
MEWMA	-	-	1.5389
Orthogonal	β_0	52.173	52.827
	β_1	6.4431	6.5569
	β_2	1.9101	2.0899
	σ	-	0.53517
dEWMA-OR	β_0	52.2908	52.7092
	β_1	6.4636	6.5364
	β_2	1.9424	2.0576
	σ	-	0.2959

Tables 2 and 3 summarized the values of in-control AARL and SDARL based on different m values. The last column shows the results when parameters are known, which mean that we can assume that an infinite (∞) number of in-control samples are available in Phase I. In-control AARL and SDARL trends for all m values using estimated parameters by F and T^2 methods are shown in Figs. 1 and 2.

Table 2. Values of in-control AARL concerning three competing methods in the case of using *m* Phase I samples for unknown parameter estimation

Control chart	Estimation method	*m*						
		30	100	500	1000	2000	5000	∞
Orthogonal	*F*	109.39	157.77	189.54	194.68	197.55	199.17	200.2
	T^2	39.42	75.60	130.70	152.93	171.40	186.33	
MEWMA	*F*	181.80	191.37	197.85	198.38	199.77	199.86	200.2
	T^2	177.31	190.38	197.32	198.07	199.51	199.53	
dEWMA-OR	*F*	92.35	145.07	185.38	193.00	196.94	199.48	199.8
	T^2	30.60	62.71	120.58	144.21	162.92	181.73	

Table 3. In-control SDARL values for three competing methods in the case of using *m* Phase I samples for unknown parameter estimation

Control chart	Estimation method	*m*						
		30	100	500	1000	2000	5000	∞
Orthogonal	*F*	41.27	26.84	8.70	5.24	3.57	2.87	2.2
	T^2	65.71	61.56	53.63	42.43	30.19	16.90	
MEWMA	*F*	51.41	37.09	18.84	13.91	10.06	7.25	3.8
	T^2	53.39	37.51	19.38	13.74	10.10	7.83	
dEWMA-OR	*F*	43.62	31.85	11.77	6.90	4.35	3.06	2.8
	T^2	65.04	60.48	56.99	47.95	36.66	21.84	

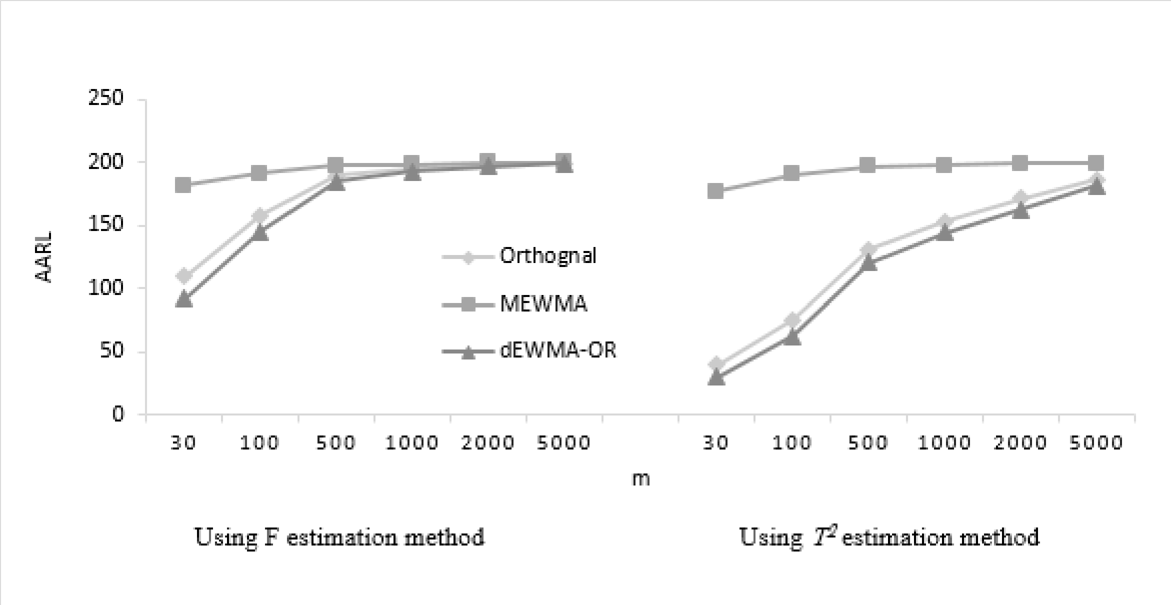

Fig. 1. AARL comparison of three rival methods concerning various m values

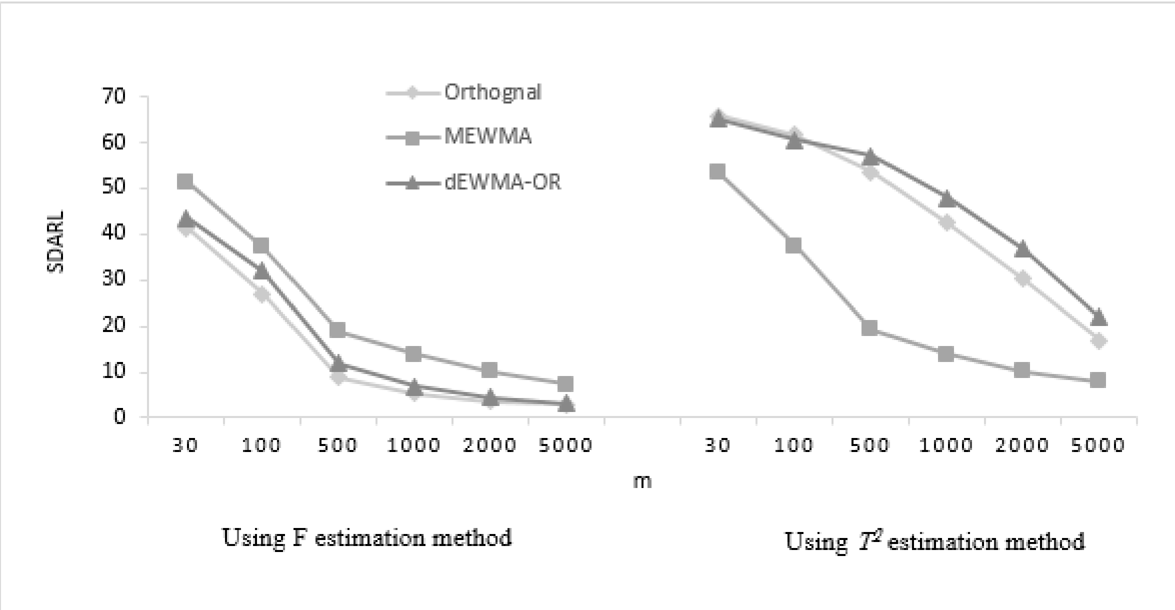

Fig. 2. SDARL comparison of three rival methods concerning various m

Results of simulations show that the in-control performance of control charts based on estimated parameters is not the same as the case of known profile parameters. So, Phase I parameter estimation strongly affects the performance of Phase II monitoring. According to Fig 1, it can be inferred that using both estimation methods, the MEWMA technique has a better performance compared to other competing approaches due to larger values of AARL.

Table 3 summarized the values of SDARL regarding various *m* values, showing that by increasing *m*, SDARL values decrease. Table 3 and Fig 2 show that Orthogonal and dEWMA-OR methods have slightly similar performance in terms of SDARL values in the case of estimating the profile parameters using the F method, however, the Orthogonal method generally gives the best SDARL performance for all *m* values. When T^2 method is applied for parameter estimation, the MEWMA chart performance is superior to other competing ones.

According to Table 2, the MEWMA method gives the best AARL performance out of the competing charts when the profile parameters are estimated by either F or T^2 approaches. It should be mentioned that the simulation studies have also been considered based on ARL metric and this result is similar to ARL-based results. Based on the results in Table 2, higher values of in-control ARL and AARL are obtained by an increase in m value. It is because of decreasing variability in the estimators sampling distribution, which occurs by an increase in m. When there is an increase in m, there is a decrease in estimation error and the values of in-control ARL and AARL come to the designated value, which justifies taking larger values of m when estimated parameters are used instead of known parameters.

The out-of-control performance

Out-of-control performance evaluation for the mentioned monitoring methods according to estimated parameters also plays a very important role in detecting the shifts (changes) in the process as soon as possible. For this aim, in the first profile generated in Phase II, according to Step 3 of the simulation algorithm presented in Section 3.1, some sustained step shifts are created in one of the parameters β_0, and σ. Note that, according to the known parameters, the out-of-control ARL tends to 1 and the SDRL tends to standard deviation of the geometric distribution.

For an out-of-control condition, the control limits should be found according to the known parameters and chart statistic should be obtained according to the estimated parameters. Therefore, under such kind of set up, an out-of-control signal is due to two variability sources, including the effect of parameter estimation plus assignable causes. To compare performance fairly, the design of the control charts has to produce a same in-control desired ARL. As previously stated, in-control ARL, using estimated parameters, is smaller than the expected one according to known parameters. As shown by the results, to achieve the specified ARL, too many Phase I samples will be required for estimating parameters, while collecting samples requires a long time. To put is another way, long waiting for data collection may result in shifts in profile parameters that will affect the performance of the in-control ARL. To solve this problem, wider control limits are applied, called “corrected limits”. The corrected limits are recently used in some previous research related to the effects of estimating parameters, including Quesenberry [51], Jones [52], Champ et al. [31] and Mahmoud and Maravelakis [53].

Corrected limits are calculated due to the parameter estimation variability. Using corrected limits for an out-of-control process ensures that the chart provides an out-of-control signal only due to the profile parameters shifts, not the estimation variability. Hence in Step 1 of the proposed simulation algorithm, the corrected limits are used for comparison of the out-of-control performance of the three rival methods. We obtained the corrected limits by 10,000 replications of the Monte Carlo simulation discussed in Section 3.1 for each monitoring method to obtain $ARL_0 \cong 200$. Tables 4-6 summarize the simulated corrected limits values according to m in-control Phase I samples.

Table 4. Corrected limits for Orthogonal method to obtain ARL = 200 when using F and T^2estimation methods for the unknown parameter estimation based on different m values

Method	Control Limits	Estimation method (Phase I)					
		F			T^2		
		m			m		
		30	100	500	30	100	500
Control chart for monitoring β_0	UCL	52.855	52.837	52.830	52.968	52.900	52.846
	LCL	52.150	52.162	52.170	52.121	52.135	52.142
Control chart for monitoring σ	UCL	0.54450	0.54170	0.53800	0.55130	0.55012	0.53998

Table 5. Corrected limits for *dEWMA-OR* method in order to achieve ARL = 200 when using F and T^2estimation methods for the unknown parameter estimation based on different m values

Parameter	Control Limits	Estimation method (Phase I)					
		F			T^2		
		m			m		
		30	100	500	30	100	500
Control chart for monitoring β_0	UCL	52.7317	52.7209	52.7130	52.7699	52.7565	52.7361
	LCL	52.2692	52.2791	52.2875	52.2201	52.2445	52.2702
Control chart for monitoring σ	UCL	0.3161	0.2980	0.2968	0.3285	0.3095	0.3062

Table 6. Corrected limits for *MEWMA* method in order to achieve ARL = 200 when using F and T^2estimation methods for the unknown parameter estimation based on different m values

Control Limits	Estimation method					
	F			T^2		
	m			m		
	30	100	500	30	100	500
UCL	1.5720	1.5501	1.5409	1.5775	1.5510	1.5410

A comparison of the out-of-control performance of rival methods has been provided concerning ARL and SDRL, and Tables 7 and 8 show the results of simulations. AARL and SDARL metrics do not differ much in choosing the superior methods. The size and type of the shifts are the same as the one used in Kazemzadeh et al. [43].

The last six rows of Tables 7 and 8 show the values of the out-of-control ARL and SDRL in the case of obtaining the charts statistic according to known parameters. The simulated results show that using estimated parameters not only affects the in-control performance but has strong impacts on the out-of-control ARL and SDRL performance of all rival approaches. Obviously, we expect that both metric decreases with an increase in the m value which agrees with numerical simulation results.

Table 7. Values of out-of-control ARL and SDRL for three rival methods in the case that β_0 changes into $\beta_0 + \delta\sigma$, when using F and T^2estimation methods for the unknown parameter estimation based on different m values

m	Phase II method	metric	Phase I method	0.1	0.2	0.9	1
30	Orthogonal	*ARL*	F	163.52	39.63	2.85	2.55
			T^2	192.18	169.30	3.19	2.82
		SDRL	F	155.92	33.42	0.84	0.70
			T^2	218.62	193.39	1.87	1.60
	MEWMA	*ARL*	F	110.76	32.40	3.10	2.76
			T^2	125.41	33.95	3.03	2.71
		SDRL	F	127.82	40.71	1.31	1.05
			T^2	149.70	40.26	0.85	0.72
	dEWMA-*OR*	*ARL*	F	105.77	29.70	4.48	4.15
			T^2	184.18	48.61	4.99	4.61
		SDRL	F	94.71	21.46	0.89	0.79
			T^2	256.98	38.62	0.91	0.79
500	Orthogonal	*ARL*	F	97.88	29.24	2.64	2.38
			T^2	131.29	35.30	2.76	2.48
		SDRL	F	91.55	23.48	0.74	0.61
			T^2	125.04	29.35	0.82	0.69
	MEWMA	*ARL*	F	94.55	29.91	2.93	2.63
			T^2	97.07	30.35	2.93	2.63
		SDRL	F	92.69	24.97	0.80	0.68
			T^2	95.02	25.69	0.80	0.67
	dEWMA-*OR*	*ARL*	F	78.83	24.05	4.25	3.96
			T^2	124.84	31.00	4.55	4.24
		SDRL	F	68.31	16.23	0.64	0.58
			T^2	115.09	22.08	0.68	0.58
∞	Orthogonal	*ARL*	-	91.47	28.32	2.62	2.35
		SDRL	-	85.70	22.37	0.73	0.59
	MEWMA	*ARL*	-	92.11	29.81	2.93	2.61
		SDRL	-	85.97	24.16	0.80	0.65
	dEWMA-*OR*	*ARL*	-	71.28	22.72	4.19	3.90
		SDRL	-	62.86	14.78	0.62	0.55

Table 7 shows the estimated values of out-of-control ARL and SDRL for various changes from β_0 into $\beta_0 + \delta\sigma$. Based on known parameters, the dEWMA-OR approach had a more acceptable performance compared to others in the detection of small shifts concerning ARL. However, by increasing the shift size, the Orthogonal method has the smallest ARL values for medium to large shifts ($\delta \geq 0.4$). In addition, it is inferred based on simulation results that the dEWMA-OR approach performs better compared to other competing methods in terms of SDRL. Applying the F estimation method and for all values of *m*, the dEWMA-OR method has the best ARL and SDRL performance compared to others for detecting small shifts. For large shift sizes, the Orthogonal method outperforms the dEWMA-OR method. On the other hand, using the T^2 estimation method, the MEWMA method has the best ARL values for small to medium size of *m*, but for large sample size (for example m = 500), the Orthogonal method outperforms the MEWMA method in detecting medium to large shifts. Moreover, MEWMA and dEWMA-OR methods perform better than the Orthogonal method based on the SDRL metric for all values of *m*. The results show that using the F method for parameter estimation in Phase I generally results in higher performance of control charts in Phase II.

Table 8. Values of out-of-control ARL and SDRL for three competing approaches in the case that $\boldsymbol{\sigma}$ shifts to $\delta\sigma$, when using F and T^2estimation methods for unknown parameter estimation based on different m values

m	Phase II method	metric	Phase I method	1.1	1.2	1.9	2
30	Orthogonal	*ARL*	F	41.11	13.15	1.79	1.63
			T^2	52.78	14.37	1.81	1.64
		SDRL	F	37.22	9.80	0.82	0.71
			T^2	49.33	10.91	0.82	0.73
	MEWMA	*ARL*	F	67.25	20.10	2.46	2.22
			T^2	80.47	22.41	2.53	2.29
		SDRL	F	87.64	21.14	0.96	0.85
			T^2	106.66	24.19	1.01	0.89
	dEWMA-*OR*	*ARL*	F	34.94	12.53	2.89	2.44
			T^2	44.05	13.68	2.95	2.72
		SDRL	F	28.77	7.02	0.79	0.71
			T^2	37.30	7.85	0.79	0.73
500	Orthogonal	*ARL*	F	40.67	13.07	1.78	1.61
			T^2	45.61	13.52	1.78	1.60
		SDRL	F	36.21	9.79	0.80	0.70
			T^2	41.10	10.24	0.80	0.71
	MEWMA	*ARL*	F	46.83	15.88	2.43	2.19
			T^2	47.69	15.86	2.41	2.18
		SDRL	F	42.61	12.02	0.93	0.83
			T^2	43.22	11.69	0.92	0.82
	dEWMA-*OR*	*ARL*	F	32.48	11.93	2.79	2.57
			T^2	36.71	12.71	2.84	2.61
		SDRL	F	25.91	6.69	0.76	0.70
			T^2	29.85	7.30	0.77	0.70
∞	Orthogonal	*ARL*	-	39.45	12.94	1.75	1.59
		SDRL	-	35.56	9.62	0.79	0.70
	MEWMA	*ARL*	-	46.32	15.71	2.39	2.17
		SDRL	-	41.08	11.63	0.93	0.81
	dEWMA-*OR*	*ARL*	-	32.33	11.85	2.77	2.56
		SDRL	-	25.00	6.61	0.76	0.69

Table 8 summarizes the out-of-control ARL and SDRL values in the case of σ changes into $\delta\sigma$. According to the results obtained, when parameters are known, the Orthogonal method performs uniformly better in out-of-control ARL except for small shifts. As evidence shows, the dEWMA-OR approach has the best performance concerning the SDRL metric.

In the case of profile parameter estimation by either F or T^2 method in Phase I, the dEWMA-OR method performs the best in out-of-control ARL to detect small Phase II shifts. Nevertheless, the Orthogonal method has a more acceptable performance in comparison to its rivals in the detection of larger shifts for all m values. Like the conditions of knowing the parameters, it is observed that the dEWMA-OR method has a better performance compared to its rivals concerning the SDRL metric. The ARL and SDRL values obtained based on the F estimation method, are smaller than those obtained for the T^2 method. Therefore, when the profile parameters are unknown, according to all the above simulation results, it is strongly recommended to apply the F method for Phase I parameters estimation.

Conclusion

Although previous studies have deeply investigated profile monitoring, evaluating the parameter estimation impacts in Phase I on the control charts performance in Phase II has been under less attention of researchers. The present study mainly aims at investigating the performance of several Phase II polynomial profile monitoring approaches in the case of unknown profile parameters that require estimation. We studied three Phase II monitoring methods, including MEWMA, Orthogonal and dEWMA-OR along with two Phase I estimation methods such as F and T^2. The performance of mentioned monitoring methods subject to in-control as well as out-of-control conditions was evaluated concerning the metrics of ARL, SDRL, AARL and SDARL.

Simulation results showed that estimating parameters seriously affects run length performance in all monitoring methods during Phase II in the case of estimating the profile parameters according to a small to moderate number of Phase I samples. According to the study findings, concerning in-control ARL and AARL, the MEWMA method has a more acceptable performance compared to other rivals, and concerning SDARL, the Orthogonal method performs totally better than all others. Under out-of-control conditions, as the comparison of Phase II approaches regarding ARL revealed, although the dEWMA-OR method performed the best in the detection of small changes in β_0 and σ, the Orthogonal method performed better in the detection of larger changes. It has been proven that applying the F method instead of the T^2 method for Phase I estimation results in better control charts performance in Phase II. In addition, it is also obvious that the performance of all methods improves by increasing m.

It is suggested to investigate parameter estimation effects on the performance of Phase II monitoring approaches in different profile types, including nonlinear profiles. Besides, proposing control charts for monitoring auto correlated polynomial profiles during parameter estimation can be an attractive subject to study in this direction.

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