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RESEARCH PAPER

Proposing a New Mathematical Model for Optimizing the Purchase of Electricity Required by Large Consumers Based on Modern Portfolio Theory: A Case Study of the Iranian Electricity Market

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Abstract

In deregulated electricity markets, the electricity consumer should optimally divide the necessary electrical energy into different markets such as cash markets with spot prices and bilateral contract markets. This study aims to design a model to optimally select the electrical energy portfolio to minimize purchase costs by considering a risk level. For this purpose, an optimization model is proposed through the modern portfolio theory (MPT), mean-variance analysis, and conditional value-at-risk (CVaR) for cost minimization and risk reduction in the electricity supply problem. The mean-variance and CVaR were used as appropriate criteria for reducing unfavorable states in decision-making under uncertainty. Moreover, an artificial neural network was employed to predict the spot prices of the energy pool and the Iran Energy Exchange (IRENEX). The simulation was based on the actual data of Iran for 2018 and 2019. The entire statistical population was analyzed due to the small number of industrial subscribers, and the proposed model was implemented and executed in MATLAB software. Different sensitivity analyses proved the efficiency of the proposed models. According to the results, if an energy purchaser evades more risks, *i.e.*, the risk evasion coefficient increases a lower ratio of the electrical energy portfolio is allocated to cash markets, especially the IRENEX. In addition, the CVaR provided electricity markets with a more stable energy allocation than the mean-variance model.

Keywords:
Portfolio Optimization,
Electrical Energy Market,
Uncertainty,
Modern Portfolio Theory (MPT),
Conditional Value-at-Risk (CVAR)

Introduction

Despite all of its advantages, the liberalization of the electricity market has resulted in some operating complexities and financial risks. On the one hand, the number of market activists and players increases with their relationships becoming more complicated in this structure; on the other hand, these players exchange large volumes of financial trading. Hence, there will be different risks such as price volatility, fluctuations in trading volumes, credit risks, and

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operating risks. In such an atmosphere, it is essential to accurately identify and analyze the effects of different risks of the market players. It will also be necessary to design and adopt specific approaches and methods for managing and eliminating these risks. The electric power industry has experienced fundamental changes worldwide over the past two decades. These changes have been classified into different categories such as deregulation, revision of laws, and restructuring. In the conventional structure of the electric power industry known as systems with a vertically integrated structure, a company used to be responsible for generating, transmitting, and distributing power. In other words, one company was exclusively authorized to generate, transmit, and distribute power for the consumers within its corporate area. In the novel structure, a company does not benefit from this intrinsic monopoly, for different sectors of the electric power industry (*i.e.*, generation, transmission, and distribution) are separated [4].

In Iran, the electric power industry has experienced four evolutionary eras known as regularization, deregulation, restructuring, and privatization. The era of regularization enhanced the managerial ownership role of the state in the electric power industry, whereas deregulation changed the laws resulting in the continuous presence of the state in this industry and leading to a legal framework for the presence of the private sector. Restructuring divided the vertical monopoly into generation, transmission, and distribution. Finally, privatization led to the process of transferring ownership to the private sector. The review of literature on Iran's electricity market indicates some signs of the abovementioned risks [3].

Some certain factors and constraints distinguish Iran's electric power industry from those of other countries in socioeconomic aspects. These factors are related to a country's electric power system, political conditions, and cultural conditions. Electric power cannot be purchased and stored for consumption, and the novel electricity market is much more variable than the commodity market. The players of this novel market are prone to considerable risks entailed by changeable market conditions. This study proposes a solution to the optimal risk management problem through project portfolio management in the deregulated electricity markets. Producers and purchasers compete over the generated power and the required power in electricity markets. They announce their price proposals to the market operator at different points in time. Cash prices entail many risks and variations which can change the reactions of market activists in competitive markets. According to an enactment passed by Iran's Council of Ministers in 2015, the subscribers with contractual power above 5 MW, who are also called big consumers, can supply their required electric power from the Iran Energy Exchange (IRENEX), the energy pool, bilateral contracts, and their power plants. If the power plants fail to supply the promised power for any reason, the subscriber's power is supplied by the national power grid. According to the laws passed by Iran's Market Regulatory Board, the power plant is obliged to compensate for the relevant expenses in the electricity market.

This study proposes a model for big consumers to help them design and supply their optimal energy portfolios through the energy pool of the wholesale market, bilateral contracts with power plants, and the IRENEX in the physical market at the minimum cost. Moreover, the input electrical energy is assumed to have a key role in the consumer's production process and accounts for a considerable part of production costs. This consumer is also assumed to purchase a substantial part of a demand from the energy pool and the IRENEX. Due to the uncertainty of electricity prices, the big consumer's ultimate goal is to minimize the expected cost of power supply along with the relevant risk of price change. The risk means that fluctuations in the power supply cost increase the risk of a heavy cost significantly. Uncertainty is considered only in the spot prices of the energy pool and the IRENEX, whereas the future prices of electricity purchase contracts are risk-free and constant. They are considered a cover for the risk. In particular, the price volatility risk can be regarded as an important risk in the spot cash electricity markets. Nevertheless, there are other sources of risk such as demand change and periodic production change which can affect the price volatility and make prices unstable. In these economic conditions, financial decision-making plays a key role, especially in portfolio selection problems. The modern portfolio theory (MPT) was used in this study to minimize the purchase costs and risks in the electrical energy supply problem. Diversification is a financial method for risk control. Based on the problem hypotheses, for diversification in the energy trade, purchases from markets with spot prices such as the IRENEX and the energy pool were considered because they are risky.

In addition, bilateral contracts were used in risk-free purchases. The mean-variance criterion (MVC) model was used as the MPT, whereas the second model was considered the conditional value-at-risk (CVaR). These models were then implemented and solved in MATLAB. The data extracted from Iran's electricity market were used to show the efficiency and practicality of the proposed model. The results of solving these models indicate what types of existing contracts should be selected to determine the optimal energy portfolio and how much energy should be purchased from the spot price market, the energy pool, and the IRENEX with respect to risk management strategies. Finally, the results of solving these models are analyzed and compared.

Literature review

Regarding the electrical energy portfolio programming and recommendations, previous studies have mainly focused on the electrical energy sale portfolio. Very few studies have been conducted on how to select the optimal electrical energy purchase portfolio from the consumer perspective [6,8,9,10,28]. Zare *et al.* [14] proposed a method for determining the big consumer’s strategy to supply the electricity demand from the market. They employed the information gap decision theory to model the price uncertainty. Conejo *et al* . [11] Proposed a technical solution to the problem of electrical power producers with big consumers. They adopted a quadratic mixed-integer mathematical model to minimize the purchase costs and limit the risk volatility caused by the price instability.

Garcia *et al*. [22] used the mean-variance and CVaR models for electrical power producers. Their results indicated that the CVaR model was more conservative than the mean-variance model and provided a more stable allocation for risky markets and cash markets.

Glensk *et al.* [20] employed the fuzzy set theory in the process of optimizing the electricity sale portfolio. They proposed a mathematical framework that could identify efficient portfolio sets to maximize the expected return on the predicted risk or minimize the risk on the expected return. Liu *et al.* [8] defined the problem of energy allocation between spot price markets and bilateral contract markets as an energy portfolio optimization problem with a risk-free asset and a risky asset. They used a quadratic programming model and historical data of the electricity market to optimize the portfolio problem. Rebennack *et al.* [12] conducted a study on the electrical energy purchase portfolio optimization problem in Germany’s electricity market. They managed to determine the extent of the requested energy demand that should be generated at the consumer’s power plant as well as the extent that should be purchased from the spot price market and the amount that should be purchased by signing contracts. This problem was formulated as a mixed-integer linear programming model in the General Algebraic Modeling System (GAMS) without considering uncertainty conditions and measuring risk. Kuhn *et al.* [25] proposed a multistage variance optimization model for management by using the multistage linear decision rules (LDRs) to reduce complexity. Their approach included limiting the decision-making rules set to random parameters. Algariv *et al.* [23] proposed a retail portfolio optimization model for future markets, other electricity markets, or a combination of markets. They developed a multifactor system to simulate energy markets and emphasized the interaction between retailers and end customers. In their optimization model, the MPT was adopted for risk determination. De Filippo *et al.* [24] proposed a nonlinear optimization approach for the electricity market dynamism to obtain the offers of tariffs. Their approach is based on a stochastic model of residential electricity consumption and a certain model of big consumers. This approach was tested on the data of Italy's energy market in which an extensive analysis was conducted on different scenarios.

Barati *et al.* [5] considered the retailer perspective and the problem of arranging contracts between suppliers and consumers to maximize profit and minimize the operating costs of the distribution system. They proposed a two-level optimization model, which can minimize the distribution system cost through scattered production and maximize the retailer profit. Kehunen *et al.* [13] stated that the conventional risk management methods would usually be inefficient in the case of an electricity retailer facing the risks of volume and price while purchasing from the wholesale market. For the electricity contract portfolio management, in this case, they developed a multistage stochastic optimization method that considered uncertainty in price and electric loads and used the CVaR for risk control along with programming steps. According to the empirical results based on the actual data, it is essential to model the price–load relationships. In addition, it was concluded that a retailer would be more sensitive without considering the risk of uncertainty in price with respect to the expected cost. Furthermore, a risk-evasive retailer would be sensitive to the mere stimulators of the expected risk. Golmohammadi *et al.* [2] addressed the green production portfolio optimization from the retailer's perspective in a competitive market environment. They formulated the uncertainty in the electricity price and generation of wind and solar power energy by using stochastic variables. They also used pre-order contracts to provide a load for customers; thus, they reduced the electricity purchase risk from the customer's perspective. They claimed that consumers could enable retailers to manage the risk and profit resulting from participation in the retail market. In their study, the problem uncertainties were formulated through random programming and time series of the autoregressive integrated moving average (ARIMA) and the Monte Carlo sampling method. In addition to the risk-free bilateral contracts in future markets, they analyzed Iran's two spot price markets called the energy pool and the IRENEX, in which customers should make purchases by considering risk management. Nazemi *et al.* [37] The main objective of this paper is to predict the price of electricity in Iran's electricity market by using a combination of fuzzy-neural network and particle swarm optimization (PSO). In this paper, past prices, past loads, working and nonworking days, day hours, and the effect of seasons in 2015 have been taken into account as the effective factors in the forecasting mechanism. The combined model is more precise than other methods like ARIMA, neural network, neural-fuzzy network, and a combination of fuzzy-neural and genetic algorithms. In the following, the process of price fluctuations has been discussed for increasing effectiveness of bidding. Results of the simulation revealed that price forecasting is much more precise with the price process mechanism.

Ahmadi *et al.* [26] used the concept of portfolio optimization to show more potential for the use of renewable energy and reduce the investment risk in the electricity sector. In their proposed model, the need for renewable energy was compared with the main scenario to calculate the amount of renewable energy based on the historical data existing in the reports of energy balance sheets and the data reported by the Ministry of Energy. The main goal of this constraint is to determine the optimal value of power production capacity in Iran. The simulation was performed under specific conditions of production costs and risks to predict the share of renewable energy (in the empirical sample of power generation). Finally, simulation and optimization were performed for portfolio risk minimization. Sun *et al.* [27] reported that China's electricity market achieved considerable modifications in 2015 and led to the full liberalization of the sales market. The electricity retailers that are now trying to adapt to the electricity market focus on portfolio optimization based on risk evaluation, something which can be performed by classifying and integrating the probable trade of electricity and sale on mid-term and local markets. In this study, a scenario was implemented to simulate the random risk variables (*i.e.* actual-time price and user demand). After that, a risk evaluation model was developed for comprehensive decision-making on the electricity investment portfolio optimization with the purpose of profit maximization. The CVaR was considered the risk evaluation index, and four combinations of electricity trade cases were evaluated in a case study. The most important business case was significantly affected by risk evasion factors in relation to the purchase scale and the expected return that would confirm the proposed model. Faia *et al.* [28] proposed a novel portfolio optimization model considering a new approach to risk management. The problem of power allocation to different markets was formulated as a classical portfolio optimization problem by regarding the market price prediction error as a part of the risk asset.

Ray *et al.* [29] A heuristic approach to portfolio optimization problem using ant colony optimization (ACO) technique centering on optimizing the conditional value-at-risk (CVaR) measure in different market conditions based on several objectives and constraints has been reported in this paper. The proposed ACO approach is proved to be reliable in a collection of several real-life financial instruments as compared to its value-at-risk (VaR) counterpart. Separi *et al.* [30] proposed a comprehensive model to determine the retailer strategy for purchasing electrical power from the wholesale and/or local market in an active distribution network. The major target of this paper is to maximize the retailer benefit concerning a tolerable risk. In order to model risks, the scenario theories are exploited and for solving the optimization problem, particle swarm optimization (PSO) has been utilized. Raeiszadeh *et al.* [38] Since there is a covariant among portfolios, there are situations in which all portfolios go high or down simultaneously, known as systemic risks. In this study, we proposed three improved meta-heuristic algorithms namely, genetic, dragonfly, and imperialist competitive algorithms to study the portfolio selection problem in the presence of systemic risks. Results reveal that our Imperialist Competitive Algorithm is superior to the Genetic algorithm method. After that, we implement our method on the Iran Stock Exchange market and show that considering systemic risks leads to a more robust portfolio selection.

Hosseini *et al.* [31] In the present study, the objective function of the RMM is formulated in a market environment in order to determine the optimal demand, incentive, and power purchased with considering some technical constraints such as incentive limits, demand limits, power purchased, and power balance. Co-evolutionary Improved Teaching Learning-Based Optimization (C-ITLBO) is applied to maximize the RMM's profit. In addition, information gap decision theory (IGDT) is applied to model uncertainty in the initial electricity price. The above-mentioned items are modeled in a multi-level formulation. Davoodi *et al.* [32] In this paper, a novel method is proposed to predict the cost of short-term hourly electrical energy based on combined neural networks. Due to the fluctuations in electricity prices during various seasons and days, these parameters do not adhere to the same pattern. In the proposed hybrid method, an evolutionary search method is used to provide an appropriate initial weight for neural network training. Given the price data changes, the price amidst the previous hour has a significant effect on the prediction of the current state.

Azadi Hematabadi *et al.* [33] presented a new analytical solution method for Supply Function Equilibrium (SFE)-based bidding strategy in electricity markets. The problem is modeled as a bi-level optimization problem; on the inner level, ISO clears the market to maximize social welfare, and on the outer level, each GenCo tries to maximize its individual welfare. The proposed method is used to solve the outer level problem using an iterative algorithm, in which LSF coefficients are parameterized. The results show that the proposed method is effective, and accurate for GenCos' strategic bidding in electricity markets compared with other optimization algorithms For this purpose, a Multiobjective problem was proposed, and the particle swarm optimization method was employed because the problem was NP-hard. Moreover, a case study was conducted on the actual data of Libya's electricity market Table 1.

compares the proposed mathematical model with previous studies in Portfolio Optimization Electricity Markets literature.

The advantages of the proposed method include increasing the shareholder equity and minimizing the market participation risk. Accordingly, this study seeks to answer two questions:

1) what goals should be taken into account to design the electrical energy portfolio?
2) How should the optimal levels of each goal be selected for practical programming?

Table 1. Literature review of the Portfolio Optimization in Electricity Markets

Author(s)	Mathematical model		Uncertainty model		Markowitz Theory		Role		Dynamic
	uncertainty	Certainty	Stochastic	Markowitz Theory	MVC	CVaR	purchaser	seller	portfolio allocation
Emanuel Canelas et al. (2021)	✓	×	✓	×	×	×	✓	×	×
Faia et al. (2021)	✓	×	✓	×	×	×	×	✓	×
Ray et al. (2019)	✓	×	×	✓	×	✓	×	✓	×
Garcia et al. (2017)	✓	×	×	✓	✓	✓	×	✓	✓
Algarvio et al. (2017)	✓	×	×	✓	×	✓	✓	✓	×
Fazıl et al. (2017)	✓	×	×	✓	✓	×	×	✓	✓
Golmohamadi et al. (2016)	✓	×	✓	×	×	×	×	✓	✓
Rocha et al. (2016)	✓	×	×	✓	✓	×	×	✓	✓
Marrero et al. (2015)	✓	×	×	✓	✓	×	×	✓	×
Rebennack el al. (2010)	×	✓	×	×	×	×	×	✓	×
Liu et al. (2007)	✓	×	×	✓	✓	×	×	✓	×
This paper	✓	×	×	✓	✓	✓	✓	×	✓

The advantage of this study over similar and previous works is that the present study considered two markets with instantaneous prices in the Iran electricity market including energy pool and energy exchange in addition to the current contracts. At the same time, the industrial consumers must manage the amount of risk. In other words, previous executive and economic research is not applicable in the Iranian electricity market and does not give a real answer. Because both markets have a significant impact on costs with spot prices and current contracts in Iran. Therefore, the model is designed to match the structure of the Iranian electricity market and its efficiency.

Research methodology

In this article, two Multi-objective mathematical models are proposed for the electricity market portfolio required by the industrial consumption problem under uncertainty. According to some article references [34,35,36] and solving the real-world problems the Genetic algorithm as the known meta-heuristic algorithm was applied.

This applied quantitative study is a mathematical descriptive work of research. It is also considered a documentary desk study in which an empirical field research design was employed to collect data from scientific databases. The data collection tools included scientific databanks, papers, and books as well as the databases of the studied organization. Furthermore, data analysis was performed in MATLAB. The entire statistical population was analyzed due to the small number of industrial subscribers. To use the numerical model and studies in Iran's electricity market, possibly actual data of historical price series were used for 2018 and 2019.

Decision-Making Framework

Consumers encounter different scenarios for the supply of electrical energy. These scenarios are different in terms of intervals in mid-term programming. Hence, consumers use bilateral contracts to partially supply the required energy. A bilateral contract is a mutual agreement outside the electricity market environment.

In this study, a consumer is assumed to face six contracts as below:

- C_1: Covering the interval from the 22nd of June to the 6th of July, this contract is made and signed at the beginning of this interval.
- C_2: Covering the interval from the 7th of July to the 22nd of July, this contract is made and signed at the beginning of this interval.
- C_3: Covering the interval from the 23rd of July to the 6th of August, this contract is made and signed at the beginning of this interval.
- C_4: Covering the interval from the 7th of August to the 22nd of August, this contract is made and signed at the beginning of this interval.
- C_5: Covering the interval from the 23rd of August to the 6th of September, this contract is made and signed at the beginning of this interval.
- C_6: Covering the interval from the 7th of September to the 22nd of September, this contract is made and signed at the beginning of this interval.

Consumers participate in electrical energy markets with spot prices to make purchases. In the energy pool market and the IRENEX, transactions are spot and price-dependent. Due to the price uncertainty of these two markets, decision-making is always accompanied by specific complexities. Prices are presented in different scenarios, each of which is related to the realization of price in the energy pool market and the IRENEX throughout all intervals. In other words, each scenario indicates one energy purchase case with its relevant probability. In this study, a three-month interval was considered along with six subintervals, each of which represents half of a summer month. Each subinterval is equal to half of a month in summer. Therefore, the programming horizon contains 93 days of summer. The main decision-making variables in this problem include the amount of energy purchased from bilateral contracts, the amount of electrical energy purchased from markets with the spot price, the energy pool market, and the IRENEX. The purchase rate is obtained from the bilateral contracts of the entire interval or that of each half of the month. This process is performed without knowing the future prices of the market.

List of Symbols

Subscripts:

i	Index of assets/risk
k	Index of intervals/scenarios of trade:
t	Index of intervals:

C Index of contracts:

Parameters:

$r_p(t)$ The return rate of a risky investment portfolio during t

$r_B(t)$ The return rate of a risk-free asset during t

r_C The return rate of a complete investment portfolio including risky and risk-free assets

A “Preference factor” is a criterion for the purchaser risk evasion in decision-making

CD_t Contracts existing during t

$P_{ck}^{c,\min}$ Lower bound of the power purchased in contract c and scenario k (*kw/hr*)

P_{tk}^{D} Amount of necessary energy in each scenario at t (*kw/hr*)

P_t^D Amount of necessary energy during t

P_o^D Minimum demand for necessary energy (*kW/hr*)

d_t Interval

l_i lower bound of CVaR applied to each asset of the portfolio

u_i upper bound of CVaR applied to each asset of the portfolio

α Confidence level

λ_{tk}^{P} Price of energy purchased from the energy pool

λ_{ctk}^{c} Price of purchased bilateral contract c in the *k*th scenario during t

λ_{tk}^{M} Price of energy purchased from the IRENEX

λ_{ctk}^{C} The nominal price of purchasing contract c in the contract set C during t in scenario k

λ_{ctK}^{CD} Final price of purchasing contract c in the contract set C for use in different intervals

T Quantity of trade intervals

m Quantity of scenarios

Variables:

$w_i(t)$ The weights of returns on assets in the mean-variance criterion model

$y(t)$ A fraction of an entire portfolio allocated to risky assets in the mean-variance criterion model

VaR Value-at-risk

ξ Maximum loss (cost) in *VaR*

$CVaR$ Conditional value-at-risk

$x(t)$ Allocation of CVaR portfolio

y_k CVaR return on each asset of the portfolio in scenario k

$P_{ck}^{C} d_t$ Value purchased from contract c in scenario k during the first interval (*kw/hr*)

P_{tk}^{P} Value purchased from the electrical energy pool in scenario k at t (*kw/hr*)

S_{cw} If contract c is selected for scenario k, this variable is 1; otherwise, it is 0

γ Coefficient of risk that describes the attitude towards risk

η_k Auxiliary variable for calculating CVaR

Functions:

$E[r_p(t)]$	Expected return on the risky portfolio in *t*
$\delta_i^2(t)$	The variance of the risky portfolio in *t*
$\delta_{ij}(t)$	The variance of an expected interval of asset *i* in *t*
$\delta^2[r_p(t)]$	Covariance of intervals of assets *i* and *j* in *t*
$U[y(t)]$	Utility function

Portfolio optimization models

Harry Markowitz was the first researcher to state the relationship between risk and return as the securities portfolio theory and converted risk into a quantitative criterion through the proposed model [3]. The modern portfolio theory (MPT) measures the risk of an asset by evaluating the interplay between risk and expected return. This theory states that an asset cannot be selected only based on the relevant specifications of an asset exclusively and that the energy portfolio purchaser should determine how every asset moves along with the other assets. In Markowitz's model, a portfolio is optimal if it minimizes risk with the expected return or it maximizes return with the given risk level. This study aims to design an optimal energy purchase portfolio by minimizing purchase costs and considering risk factors. For this purpose, the mean-variance criterion model and the conditional value-at-risk (CVaR) are employed.

Mean-Variance Criterion Model

In this model, investors are assumed to be risk-evasive and aware of the return, variance, and covariance of assets. It is also assumed that the return on assets follows a formal distribution. This model is solved like a mathematical programming problem. A portfolio contains a combination of assets, *n* items of which are risky assets, whereas *n+1* items are risk-free assets. The distribution variance can be used to determine the rate of risk. Variance is the main principle in the mean-variance criterion model which was used by Markowitz in the MPT for portfolio selection under uncertainty [20].

Given the expected rate of return on each risky asset r_i, the weighted mean of return on an asset with investment ratios as weight is shown as $w_i(t)$, and the expected return on the risky portfolio r_p is shown in Eq. 1 [6], in which *t* denotes the time horizon intervals.

$$r_p(t) = \sum_{i=1}^{n+1} w_i(t) r_i(t) \tag{1}$$

The expected return and variance can be obtained, and the distribution variance can be used to determine the risk. The expected return on portfolio and variance can be determined by Eq. 2 [6].

$$E\left[r_p(t)\right] = \sum_{i=1}^{n+1} w_i(t) E\left[r_i(t)\right] \tag{2}$$

$$\sigma^2\left[r_p(t)\right] = \sum_{i=1}^{n+1}\sum_{j=1}^{n+1} w_i(t) w_j(t) \sigma_{ij}(t) \Rightarrow \sigma^2\left[r_p(t)\right] = \sum_{i=1}^{n+1}\left[w_i(t)\right]^2\left[\sigma_i(t)\right]^2 + \sum_{i=1}^{n+1}\sum_{j\neq 1} w_i(t) w_j(t) \sigma_{ij}(t) \tag{3}$$

Where $[\sigma_i(t)]^2$ and $\sigma_{ij}(t)$ denote the variances of return on capital *i* and covariance of expected return on different assets *i* and *j* during intervals of *t*. Eq. 2 indicates the expected return on possible portfolios. As Liu concluded [4], an efficient boundary can be formed by minimizing Eq. 3 in terms of weights. Solving this problem leads to no unique answers but results in a set of points called the efficient boundary. The direct line connects the return on a risk-free asset to each point of the efficient boundary. It is called the energy allocation line (EAL). The point of tangency on the efficient boundary is called the preference point, at which the EAL is tangent on the efficient boundary with the highest slope. At that point, the ratio of reward to risk is at the maximum. Fig. 1 demonstrates the efficient boundary and point of tangency on the EAL. The EAL includes all the possible combinations of risk and returns resulting from different allocation options. Encountering the EAL, an investor should select the optimal investment portfolio c from a set of possible options. Unlike the investors who embrace risk, a more risk-evasive investor invests less often in risky assets and more often in risk-free assets.

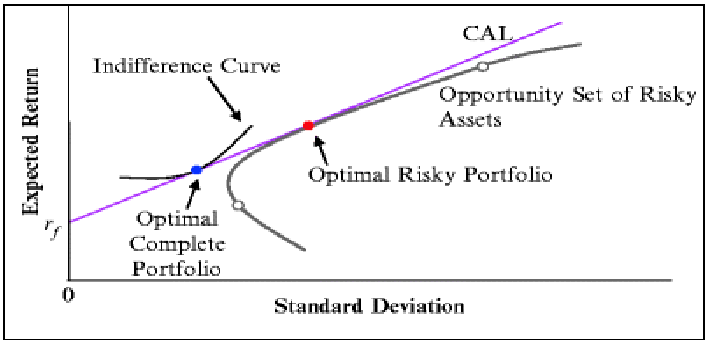

Fig. 1. Selecting the optimal asset portfolio [3]

The investor who purchases a risky portfolio with a risk-free return rate r_B, the expected return $E(r_p)$, and standard deviation σ_p will obtain the expected return on the entire investment portfolio for every ratio by Eq. 4 [6].

$$E(r_c) = r_B + y\left[E(r_p) - r_B\right] \tag{4}$$

The total variance of the portfolio is calculated by Eq. 5 [6]:

$$\sigma^2{}_c = y^2\sigma_p^2 \tag{5}$$

In Markowitz's theory, investors try to maximize their utility by selecting the best allocation to a risky asset. The proposed utility function is defined as in Eq. 6 [6]:

$$U = E(r_c) - 1/2A\left[\delta(r_c)\right]^2 \tag{6}$$

Where U and r_c denote the utility value of the "utility and preference function", and r_c refers to the expected return on a completely risky portfolio. Moreover, A indicates a criterion for the risk evasion of an investor, and the ½ factor is merely a contractual scale [16].

Eq. 6 is consistent with the concept that increasing the expected return will increase utility which decreases by increasing risk. A risk-free portfolio creates a utility score equal to a specific return rate, for the portfolio receives no fine for risk. The utility reduction caused by the variance of risky investment portfolios depends on the risk evasion of investor A, and more risk-evasive investors with larger values of A fine risky portfolios more severely; therefore, investors will select a portfolio with the highest utility. According to previous studies, the values of A can be considered from 1 to 5, and the problem of utility maximization of assets can be defined as Eq. 7 [16,20].

$$\underset{w_i(t)}{Max}\ U = E(r_c) - \frac{1}{2} A\sigma_c^2 \tag{7}$$

$$s.t: \quad \sum_{i=1}^{n} w_i(t) = 1 \tag{8}$$

$$W_i(t) \geq 0 \qquad [i+1,...N] \quad \forall t \tag{9}$$

Eq. 8 indicates that the total weights of assets in a portfolio must be equal to 1, whereas Inequality (9) depicts that the share of each asset in the portfolio cannot be negative. In other words, short selling is impossible. An optimal portfolio is determined with respect to the nature of the problem and the weights of assets. The allocation of assets for the selection of an optimal portfolio is a quadratic programming problem, which can be solved either directly through a standard quadratic programming algorithm or a software suite in MATLAB. This problem is divided into two sub-problems, in the first of which the return on a completely risky portfolio will be calculated. The second step includes the optimal allocation and integration of risky assets with risk-free assets. This is the standard two-step solution approach to portfolio optimization. Liu has proven the portfolio optimization theorem [7]. The optimal position in a risky asset (y^*) is determined by making the derivative of this expression zero in Eq. 7.

Conditional Value-at-Risk model (CVaR)

In the MPT, the risk is defined as the changeability of all returns around the mean and is calculated through the variance criterion. If the distribution is assumed normal, the variance will be an acceptable criterion for measuring the return risk; however, the real-world studies and theoretical contexts reject this assumption [3]. Therefore, when the returns follow an asymmetric distribution, variance is an appropriate criterion for risk evaluation because it fines appropriate upward trends in price as much as inappropriate downward trends in price. In reality, a reasonable short-term investor not only seeks and welcomes positive fluctuations in price but also looks for a way to measure negative fluctuations in the portfolio to select the optimal portfolio with the minimum unfavorable risk based on the results [17,18].

This approach is considered the main tool for risk measurement and risk management. The value at risk is the highest loss that a portfolio is expected to experience in a predetermined time horizon at a specific confidence level. A major advantage of this tool is to summarize risks into a single number [15]. Unlike the simply understandable concept of VaR, its calculation has many difficulties. In other words, VaR denotes the maximum loss at the confidence level of (1-α) during a specific period. In this model, risk occurs when the daily loss exceeds VaR, and the probability that the realized loss deviates from the determined VaR will be ξ. The VaR measurement model determines how confident an investor is about his/her portfolio at α that the loss will exceed Y rials during a future period (T). The value of Y can be obtained from Eq. 10 [22].

$$Y = -CDF_v^{-1}(1-\alpha) \tag{10}$$

Where CDF_v^{-1} is the inverted cumulative distribution function for investment profit V, and α is the investor's confidence level. Statistically, calculating VaR means finding the critical value of the intended probability level [3]. Since the return probability distribution is not constant over time, calculating VaR will face some problems, a major one of which is the incoherence of this criterion. In recent years, CVar has been introduced to modify and complete VaR. The conditional value-at-risk estimates the expected loss equal to or higher than VaR at the confidence level. Hence, this method is more conservative than the previous one. Given the precautionary aspect of CVaR and its widespread use in recent years, this study focuses on this criterion as a risk index. Unlike the VaR model, the CVaR measurement model has a coherent criterion that is characterized by normality, collectability, positive homogeneity, and equal transfer. However, the collectability index is a mental principle of any investor. Due to the lack of this index, the VaR model is not considered coherent. For instance, the collectability rule plays a central role in the capital sufficiency requirements of banks from a supervisory perspective. Consider the branches of a bank. If the capital requirements of each branch are determined with respect to its risk, a supervisor can ensure that the total capital of all branches will be sufficient in accordance with the collectability rule. However, based on the VaR criterion, the total risk is equal to the summation of risks at all branches. This criterion was developed in a paper by Auckerby and Tasche to cover the coherence indices [3].

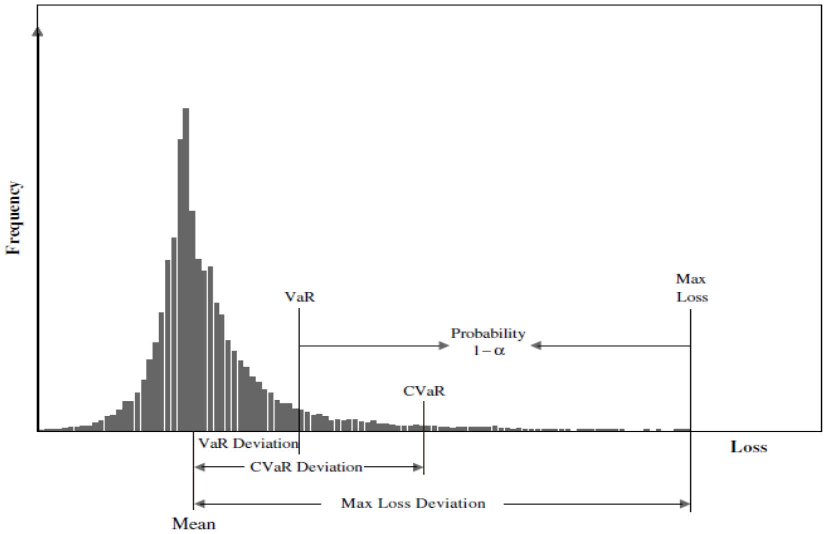

Fig. 2. Value-at-risk and conditional value-at-risk [3]

Finally, the VaR method is unfavorable due to the lack of sub-collection, which makes calculation difficult during the implementation of scenarios, and also due to the presence of a nonconvex and uneven function with multiple values of local extrema. The CVaR criterion is characterized by sub-collection, convexity, and evenness. It is more appropriate than VaR because it considers more losses. Fig. 2 demonstrates the positions of VaR and CVaR [3]. Eq. 11 [22] indicates that the value of CVaR measures the expected loss when it exceeds VaR.

$$CVaR = E(Loss_V \setminus Loss_V > VaR) \tag{11}$$

If f(x, y) in Eq. 11 [22] denotes the loss related to the decision vector x and the random vector y, then x can be shown by a portfolio. Furthermore, y indicates the uncertainties affecting the loss. As a result, the return on portfolio x is the summation of returns on every single capital in a portfolio on a scale of x_j. Since the loss denotes the negative expected return, it is defined as Eq. 12 [6]:

$$f(x,y) = -[x_s y_s + x_c y_c] = -x^T y \tag{12}$$

Where x_S denotes the ratio of a portfolio that has a risky asset in the cash market, and x_C is used in a risk-free asset. The performance function is defined in Eqs. 13 and 14 based on CVaR [22].

$$CVaR = F_a(x,\xi) = \xi + (1+a)^{-1} \int_{y \epsilon R} w[f(x,y) - \xi]^+ p(y)dy \tag{13}$$

$$\psi(x,\xi) = \int_{f(x,y) \leq \xi} p(y)dy \tag{14}$$

Where *p(y)* is the density function *y*, and the cumulative distribution function $\psi(x,\xi)$for loss is related to *x*. It is also assumed that ξ denotes the value-at-risk for a specific portfolio at the confidence level α. In the above equation, F_a is an approximation obtained from the Monte Carlo simulation, which $F_a(x,\xi)$ is defined as an approximation of F_a through probabilistic distribution sampling in *y* in Eq. 11 [22]:

$$F_a(x,\xi) = \xi + \frac{1}{w(1-a)} \sum_{w=1}^{w} [f(x,y) - \xi]^+ \tag{15}$$

Where *m* denotes a sample number *p(y)*. The estimated function $\tilde{F}_\alpha(x,\xi)$ is convex. Minimized by linear search techniques or a primary programming problem, this function is linear and segmented ξ. Since the risk of uncertainties should be considered in the decision-making problem to supply energy, this paper uses the CVaR at the confidence level α to model the risk entailed by changes in costs. The CVaR is the expected value of $(1-a) * 100\%$, the scenarios with the largest value of cost. Mathematical Model (16) indicates the CVaR [19-35].

$$Cvar = min\xi + \frac{1}{w(1-a)} \sum_{w=1}^{w} \pi_w \eta_w \tag{16}$$

To delete the nonnegative constraints of the above function, an auxiliary variable called $\eta_k\ (k = 1, \ldots, m)$ is added to the model along with other constraints. Eqs. 17 and 18 indicate this process.

$$\eta_k \geq 0 \tag{17}$$

$$x^\tau y_k + \xi + \eta_k \geq 0 \tag{18}$$

Eventually, the risk of uncertainties should also be considered in the decision-making problem to supply energy. In this paper, the CVaR is used at confidence level α to model the risk caused by the risk of changes in costs. The optimization of CVaR and the expected return on a production asset are obtained from the general optimization model [19].

Mathematical model

This section proposes a multi-objective mathematical model through Markowitz's MPT based on the mean-variance criterion and conditional value-at-risk for the dynamic allocation of an electrical energy portfolio.

Problem Description

Based on the modern portfolio theory (MPT), this model addresses the electrical energy supply problem to minimize purchase costs and risk. Diversification is a financial method for risk control. Based on the problem hypotheses, purchases were made from the IRENEX and the energy pool for diversification in energy trades because they are considered risky purchases, whereas bilateral contracts were used for risk-free purchases. For this purpose, two models of the modern portfolio theory were described and developed for portfolio optimization. The MPT measures and indicates the risk of an asset by evaluating the exchange between the expected return and risk. An asset cannot be considered merely based on its expected return, and purchases should analyze the energy portfolio to determine how an asset moves along with all the other assets at the same time. In Markowitz's model, a portfolio is optimal if it entails the minimum risk in an expected return or the maximum return at the given risk level. In this model, an optimal energy portfolio is designed to minimize the costs of purchase by considering the risk factors and using the mean-variance criterion model and the conditional value-at-risk.

Optimal Portfolio Allocation Algorithm through the Mean-Variance Criterion Model

The mean-variance criterion (MVC) algorithm consists of the following steps in the optimal electrical energy portfolio allocation problem:

- 1 – The system input data: the input price data of a market
- 2 – Predicting the forthcoming prices: using artificial neural networks to predict the forthcoming prices
- 3 – Determining the variance, covariance, and return through the sets of inputs and predicted prices: Calculate the variance, covariance, and return.
- 4 – Determining the weights of portfolio components: solving the problem through Eq. 19 and calculating the portfolio components
- 5 – Determining the allocation of portfolio components: solving the optimization problem through Eqs. 36 and 38 and determining the portfolio components through Eqs. 41 and 42
- 6 – Determining the portfolio allocation for both studied periods: repeating Step 2 to Step 5 and determining the allocation of portfolio w(i, t) for every period of the programming horizon

Step 1: The completely risky optimal electrical energy portfolio for the MVC model is proposed below to determine the optimal weights based on the nature of the model costs.

$$\underset{w_i(t)}{Max} \quad (-s) = \frac{r_B - E[r_p(t)]}{\sigma[r_p(t)]} \tag{19}$$

The return rate of the risky portfolio and the standard deviation of Eq. 19 are determined through Eqs. 20 and 21.

$$E[r_p(t)] = \sum_{i=1}^{n} w_i(t) E[r_i(t)] \tag{20}$$

$$\sigma[r_p(t)] = \left(\sum_{i=1}^{n} \left[w_i(t)\right]^2 \left[\sigma_i(t)\right]^2 + \sum_{i \neq j} \sum w_i(t) w_j(t) \sigma_{ij}(t) \right)^{1/2} \tag{21}$$

$s.t:$

$$\sum_{i=1}^{n} w_i(t) = 1 \tag{22}$$

$$W_i(t) \geq 0 \qquad \forall t \tag{23}$$

$$P_{tk}^{s} + P_{tk}^{P} + \sum_{c\in CDt} P_{ck}^{C} d_t = P_{tk}^{D} \qquad \forall t, \forall k \tag{24}$$

$$P_{tk}^{D} \geq P_{O}^{D} \qquad \forall t, \forall k, \quad \forall k \in m \tag{25}$$

$$0 \leq P_{ck}^{C} d_t \leq P_{c}^{C,\max}; \forall c \in C, \quad \forall t \in CD_t \tag{26}$$

$$P_{ctk}^{c} = 0, \quad \forall c \in C \quad , \forall t \in T \backslash CD_t \quad , \forall k \in m \tag{27}$$

$$P_{c}^{c,\min} Sck \leq \sum_{c\in CD_t} P_{ck}^{c} d_t \leq P_{c}^{C,\max} Sck \qquad \forall c, \forall k \tag{28}$$

$$Sc_m = Sc_{m+1} \qquad \forall c \ , \ m \in Z \qquad If \ \ sim \quad (m,k) = 1 \tag{29}$$

$$P_{cm}^{C} = P_{c_{m+1}}^{C} \qquad \forall c \ , \ m \in Z \qquad If \ \ sim \quad (m,k) = 1 \tag{30}$$

$$r_B = \frac{\sum_{t\in T} \sum_{c\in CD_t} \lambda_{ctk}^{C} P_{ck}^{C} d_t - \sum_{t\in T} \sum_{c\in CD_t} \lambda_{ctk}^{CD} P_{ck}^{C} d_t}{\sum_{t\in T} \sum_{c\in CD_t} \lambda_{ctk}^{CD} P_{ck}^{C} d_t} \qquad \forall t, \forall c, \tag{31}$$

$$r_i = \frac{\sum_{t=1}^{T} \lambda_{(t+1)k}^{P} \; P_{(t+1)k}^{p} - \sum_{t=1}^{T} \lambda_{tk}^{P} P_{tk}^{P}}{\sum_{t=1}^{T} \lambda_{tk}^{P} P_{tk}^{P}} \tag{32}$$

$$r_j = \frac{\sum_{t=1}^{T} \lambda_{(t+1)k}^{s} \; P_{(t+1)k}^{s} - \sum_{t=1}^{T} \lambda_{tk}^{s} P_{tk}^{s}}{\sum_{t=1}^{T} \lambda_{tk}^{s} P_{tk}^{s}} \tag{33}$$

$$P_{tk}^{P} \ , \ P_{tk}^{s} \geq 0 \quad \forall t \ , \forall k \tag{34}$$

$$S_{cm} \in \{0,1\} \ \forall c \ , \forall m \tag{35}$$

According to Eq. 19, the optimal ratio in a risky asset had an inverse relationship with the risk evasion level and the risk level measured by variance. However, it has a direct relationship with the mere risk-on assets. There is no risk in the value of expected r_B in the risk-free assets. Eq. 21 indicates the standard deviation of a risky portfolio during t, whereas Eq. 22 depicts that the summation of weights of assets in a portfolio will be equal to 1. Moreover, Inequality (23) indicates that the share of each asset in the portfolio cannot be negative. In other words, short selling is not possible. Constraint (24) guarantees that the necessary energy is provided in all intervals and scenarios, whereas Constraint (25) indicates that the required demand must be greater than or equal to 5 MW/hr. Moreover, Constraint (26) determines the range of energy consumed in each contract in each interval, and Constraint (27) indicates that it is impossible to purchase energy outside the programming horizon of each contract.

Constraint (28) determines the upper bound and the lower bound on the energy consumed by contracts in each subset of intervals. Constraints (29) and (30) model the unpredictable constraints, respectively. Each scenario contains a possible solution. During a programming period when the scenarios are realized equally, the values of decision variables are equal in this step. In other words, this constraint is employed to limit the decision-making variables related to a node with equal values in different scenarios as an origin. In this study, purchasing a bilateral contract means a risk-free trade that can be placed in combination with the portfolio based on Table 1. Its rate of return (r_B) is calculated through Eq. 31. Since it is risk-free, it is concluded that E(r_B)= r_B and $\delta^2 E(r_B) = 0$, in which λ_{ctk}^{C} denotes the price of contract c in scenario k at *t*, whereas P_{ctk}^{C} denotes the energy purchased from contract c in scenario k during the contract period for t. In addition, λ_{ctk}^{CD} is the certain price during the first period of contract c in scenario k at t. The expected return during the contract period for purchase from the energy pool is obtained from Eq. 32, whereas the return on purchase from the IRENEX is obtained from Eq. 33 in spot-price markets. Eq. 34 indicates the nature of decision variables in the model. It shows three binary variables and determines whether contract c is selected in step m. If this occurs, the variable takes a value of 1; otherwise, its value is 0. In this step of solving the risky optimal portfolio optimization problem, the value of W_i^* is determined. This equation indicates that the optimal ratio in a risky asset had inverse relationships with the risk evasion level and the risk level measured by variance; however, it has a direct relationship with the mere risk of an asset.

Step 2: Determining the optimal allocation between risky and risk-free assets based on the MVC model. An energy purchaser selects the best allocation to a risky asset to achieve utility based on the reduction of purchase costs and risk. Therefore, the utility function in Eq. 6 is considered the minimum in Eq. 36. The value of U(y) indicates the utility function and preferences, whereas r_c denotes the expected return of a completely risky portfolio. Moreover, *A* is a criterion for risk evasion. According to previous studies, the values of *A* can be considered from 2 to 4 [22].

$$\underset{y(t)}{Min}\, U(y) = E[r_c(t)] + \frac{1}{2} A\sigma^2[r_c(t)] \tag{36}$$
$$s.t.$$
$$E\left[r_c(t)\right] = \left[1 - y(t)\right] r_B + y(t) E\left[r_p^*(t)\right]$$
$$\sigma^2 r_c(t) = y(t)^2 \sigma^2 [r_p^*(t)]$$

As a result:

$$\underset{y(t)}{Max}\, U(y) = E\left[r_c(t)\right] + \frac{1}{2} A\sigma^2 \left[r_c(t)\right] = r_B + y(t)\left[E[r_p^*(t)] - r_B\right] + \frac{1}{2} Ay(t)^2 \sigma^2 [r_p^*(t)] \tag{37}$$

Where y(t) is the fraction of a complete portfolio allocated to risky assets in comparison with [1-y(t)] allocated to risk-free assets. In this step, the optimal allocation of risky assets (y*) is calculated by determining the derivative of this equation and putting it zero. The equations are as Eqs. 38 to 42.

$$y^* = \frac{r_B - E[r_p(t)^*]}{A\delta^2[r_p(t)^*]} \tag{38}$$

$$E\left[r_p(t)^*\right] = \sum_{i=1}^{n} w_i^*(t) E\left[r_i(t)\right] \tag{39}$$

$$\sigma^2\left[r_p(t)\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i^*(t) w_j^*(t) \sigma_{ij}(t) \tag{40}$$

$$Risk - free \quad asset \quad ; \overline{w}_{n+1}(t) = 1 - y^*(t) \tag{41}$$

$$Risk \quad asset \qquad \bar{w}(t) = y^{*}(t) w_i^{*}(t) \qquad i = 1,....n \qquad and\ t = 1,...,93 \tag{42}$$

Optimal Portfolio Allocation Algorithm through CVaR Model

The following steps and Eqs. 43 to 57 present the multi-objective optimization model of the optimal portfolio allocation problem through the CVaR method:

- 1 – The system input data: the input price data of a market
- 2 – Predicting the forthcoming prices: using artificial neural networks to predict the forthcoming prices
- 3 – Determining the variance, covariance, and return through the sets of inputs and predicted prices: Calculate the variance, covariance, and return.
- 4 – Determining the optimal allocation between risky and risk-free assets: solving the optimization problem through Eq. 43 and determining the allocation of assets in the energy portfolio X(t)'s

$$MinU = \left[\frac{1}{m}\sum_{k=1}^{m} x(t)^{\tau} y(t)_k\right] + \gamma\left[\xi(t) + \frac{1}{m(1-\alpha)}\sum_{k=1}^{m}\eta(t)_k\right] \tag{43}$$

$$\left[\frac{1}{m}\sum_{k=1}^{m} x(t)^{\tau} y(t)_k\right] = \frac{1}{m}\sum_{k=1}\sum_{t=T}\left(\sum_{c\in CD_t} x_{ctk}^{C} y_{ck}^{c} d_t + x_{tk}^{s} y_{tk}^{s} + x_{tk}^{p} y_{tk}^{p}\right)$$

st :

$$P_{tk}^{S} + P_{tk}^{P} + \sum_{c\in CDt} P_{ck}^{C} d_t = P_{tk}^{D} \qquad \forall_t, \forall_k \tag{44}$$

$$P_{tk}^{D} \geq P_{o}^{D} \qquad \forall_t, \forall_k, \quad \forall k \in m \tag{45}$$

$$0 \leq P_{ck}^{C} d_t \leq P_{c}^{C,\max}; \forall c \in C, \quad \forall t \in CD_t \tag{46}$$

$$P_{ctk}^{C} = 0, \quad \forall_c \in C \quad , \forall_t \in T \backslash CD_t \quad , \forall_k \in m \tag{47}$$

$$P_{c}^{c,\min} Sck \leq \sum_{c\in CD_t} P_{ck}^{c} d_t \leq P_{c}^{C,\max} Sck \qquad \forall_c, \forall_k \tag{48}$$

$$x(t)^{\tau} y(t)_k - \xi(t) \leq \eta(t)_k \quad \forall k \in m \quad k = 1,...,m \tag{49}$$

$$\sum_{t=T}\left(\sum_{c\in CD_t} x_{ctk}^{C} y_{ck}^{c} d_t + x_{tk}^{s} y_{tk}^{s} + x_{tk}^{p} y_{tk}^{p}\right) - \xi(t)_k \leq \eta(t)_k \quad \forall_k \in m \quad k=1,...,m \tag{50}$$

$$\eta(t)_k \geq 0 \qquad \forall_k \in m \qquad k = 1,...,m \tag{51}$$

$$l_i \leq x(t)_i \leq u_i, i = 1,...,n \quad for \quad t = 1,...,31 \quad day \tag{52}$$

$$\sum_{i=1}^{n} x(t)_i = 100\% \tag{53}$$

$$P_{tk}^{p}, \ P_{tk}^{s}, \zeta, \eta_k \geq 0 \quad \forall_c, \forall_t, \forall_k \tag{54}$$

$$Sc_m = Sc_{m+1} \qquad \forall c \ , \ m \in Z \qquad If \ \ si\,m \quad (m,k) = 1 \tag{55}$$

$$P_{cm}^{C} = P_{c_{m+1}}^{C} \qquad \forall c \ , \ m \in Z \qquad If \ \ sim \quad (m,k) = 1 \tag{56}$$

$$S_{ck} \in \{0,1\} \ \forall_c \forall_k \tag{57}$$

- The expected values of return on costs in the components of each portfolio are determined through Eqs. 58 to 60 based on daily changes (y_k).

$$y_{ck}^{C}d_t = \frac{\sum_{t\in T}\sum_{c\in CD_t}\lambda_{ctk}^{C}P_{ck}^{C}d_t - \sum_{t\in T}\sum_{c\in CD_t}\lambda_{ctk}^{CD}P_{ck}^{C}d_t}{\sum_{t\in T}\sum_{c\in CD_t}\lambda_{ctk}^{CD}P_{ck}^{C}d_t}\ \forall t, \forall c, \tag{58}$$

$$y_{tk}^{P} = \frac{\sum_{t=1}^{T}\lambda_{(t+1)k}^{P}\quad P_{(t+1)k}^{P} - \sum_{t=1}^{T}\lambda_{tk}^{P}P_{tk}^{P}}{\sum_{t=1}^{T}\lambda_{tk}^{P}P_{tk}^{P}} \tag{59}$$

$$y_{tk}^{s} = \frac{\sum_{t=1}^{T}\lambda_{(t+1)k}^{s}\quad P_{(t+1)k}^{s} - \sum_{t=1}^{T}\lambda_{tk}^{s}P_{tk}^{s}}{\sum_{t=1}^{T}\lambda_{tk}^{s}P_{tk}^{s}} \tag{60}$$

The following parameters must be obtained to solve the CVaR optimization model.

- The parameter γ indicates the importance of risk in comparison with goal realization (γϵ[0, 10]).
- L_i *and* u_i $\epsilon[0,1]$ Denote the lower and upper bounds of each component in the portfolio.
- The values for all scenarios are considered m (m= 64).
- The confidence level is α= 0.95.
- The variances of each component in the portfolio are necessary for the Monte Carlo simulation.

Not only is the utility function obtained during the implementation of this optimization problem, but the values of VaR and CVaR will also be determined and shown as VaR= $\xi(t)$ and $CVaR = \left[\xi(t) + \frac{1}{m(1\text{-}\alpha)}\sum_{k=1}^{m}\eta(t)_k\right]$, respectively.

Including costs and risks, Eq. 43 seeks to minimize costs by selecting the possible scenarios. Making purchases from the spot-price markets will be risky, whereas making purchases from bilateral contracts will be risk-free. In this equation, the CVaR was considered at the confidence level of α= 0.95, and β is the risk coefficient that describes the attitude towards the risk level. It is a number that strikes a balance between the expected value of cost and risk. Depending on the consumer priorities, it is a weight factor that balances the expected costs of purchase and risk ($\beta\epsilon[0, 10]$). Constraint (44) guarantees that the necessary energy is provided in all intervals and scenarios, whereas Constraint (45) indicates that the required demand should be greater than or equal to 5 MW/hr. Furthermore, Constraint (46) determines the range of energy consumed in each contract during every interval, whereas Constraint (47) states that it is impossible to purchase energy outside the programming horizon of every contract. Constraint 48 determines upper and lower bounds for energy consumption in contracts during every subset of intervals. Constraints (49) to (51) provide the constraints of the calculation of CVaR. Eq. 52 defines L_i and U_i as the lower and upper bounds of CVaR, respectively. They must be applied to every asset of the portfolio. Eq. 53 indicates the weights of assets in every portfolio that must be equal to 1. In other words, the total percentage of assets in a portfolio should be 100%. Eq. 54 depicts the nature of decision variables in the model. Moreover, Constraints (55) and (56) model the unpredictable constraints of the model. Eq. 57 indicates the binary variable showing whether Contract C is selected in scenario k. If this occurs, the variable is 1; otherwise, it is 0.

Use of MPT in Iran's electricity markets

The users who consume more than 5 MW of electricity participate in spot-price electricity markets because the price of electricity plays a central role in the net price of the product. Therefore, the less costly and low-risk the portfolio, the more appropriate and attractive it will be to consumers. In the energy pool market and the IRENEX, transactions are spot and price-dependent. Due to the uncertainty of price in these two markets, decision-making is always accompanied by specific complications. Prices are expressed in different scenarios, each of which is related to the realization of price in the energy pool and the IRENEX over all intervals. In other words, each scenario indicates a case of energy purchase with a specific probability. In this study, a three-month interval was considered with six subintervals, each of which indicates half of a summer month. The main decision variables were to determine the amount of energy purchased from bilateral contracts and the amounts of electrical energy purchased from spot-price markets, the energy pool market, and the IRENEX. In all intervals, the amount of purchase from bilateral markets pertained to the entire interval or half of a month. This process was performed without knowing the future prices of the market. Fig. 4 reports different costs of purchasing electrical energy from the energy pool, the IRENEX, and their mean in various intervals. Fig. 3 demonstrates different costs of purchasing energy from contracts in various intervals. Fig. 5 indicates the demand for energy purchase in different intervals for electrical energy demand both on a daily basis and on every interval. Table 2 reports the price (minimum and maximum) demand delivered in every interval of the programming horizon of energy supply contracts.

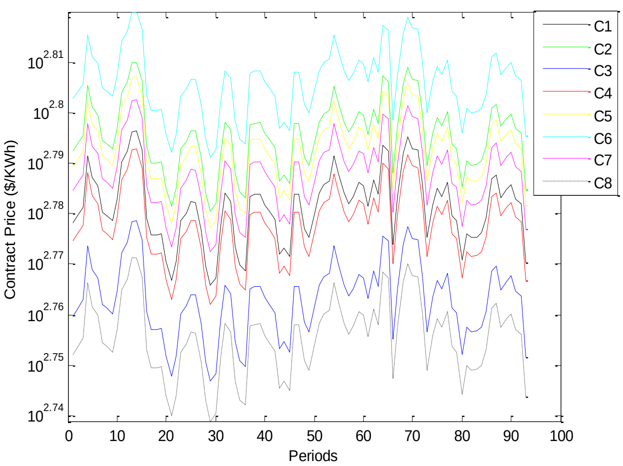

Fig. 3. Different costs of purchasing energy from contracts during various intervals

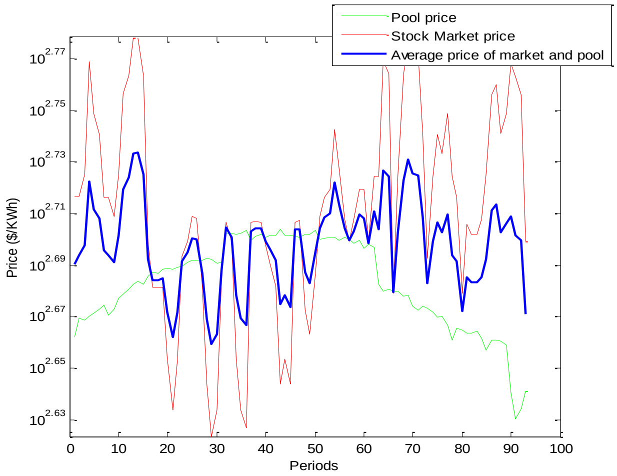

Fig. 4. Different costs of purchasing energy from the pool, IRENEX, and their mean during various intervals

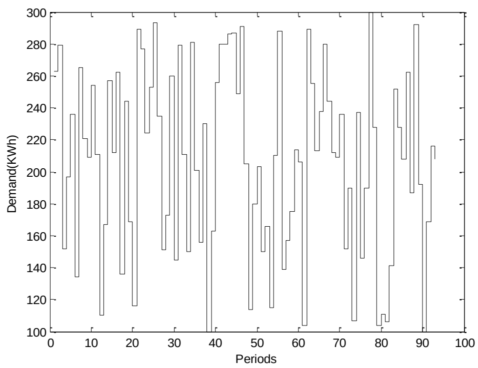

Fig. 5. Energy purchase demand during different intervals

Table 2. Energy supply contracts

C		t	$P_c^{C,max}$	$P_c^{C,min}$	λ_{ctk}^{C}	λ_{ctk}^{CD}
		Day	kW/hr	kW/hr	Rial/kw	Rial/kw
June	1	1 - 15	2500	200	658.7	660
	2	16 - 31	3500	500	698.6	700
July	3	32 - 46	4000	500	739.5	741
	4	47 - 62	4000	300	778.4	780
August	5	63 - 77	3500	200	726.5	728
	6	78 - 93	2500	600	638.7	640

Multi-objective Mathematical Model Based on Mean-Variance Criterion

The optimization techniques were adopted in the execution of this model. The standard deviation of a normal distribution is assumed to be an appropriate criterion for risk. The optimization algorithm was executed through Tool Boxes in MATLAB to determine the optimal portfolio. The first step is to determine the opportunities of risk and return available to the energy purchasers based on the characteristics of risk purchase sources. These opportunities are summarized with the minimum variance boundary of risky assets. This boundary is the indicator of the minimum possible variance that can be obtained with respect to the expected return on the portfolio. According to the input data Figs. 3 and 4 regarding the expected return, variance, and covariance, the portfolio with the minimum variance was determined for every expected return. Fig. 6 demonstrates the diagram of the standard deviation and expected return. All of the investment portfolios existing on the minimum variance boundary provide the best combinations of risk and expected return; hence, they substitute the optimal investment portfolio selection and constitute the efficient boundary of risky assets.

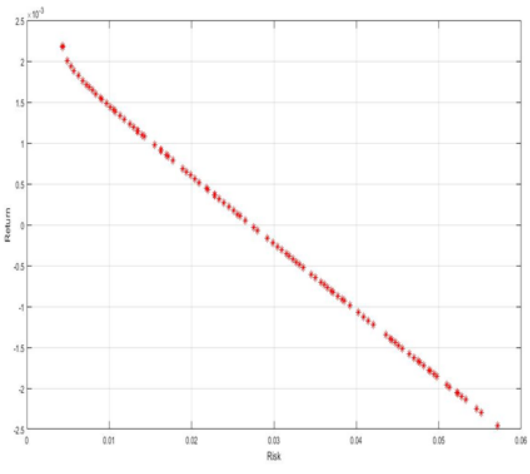

Fig. 6. The efficient boundary of risky purchases

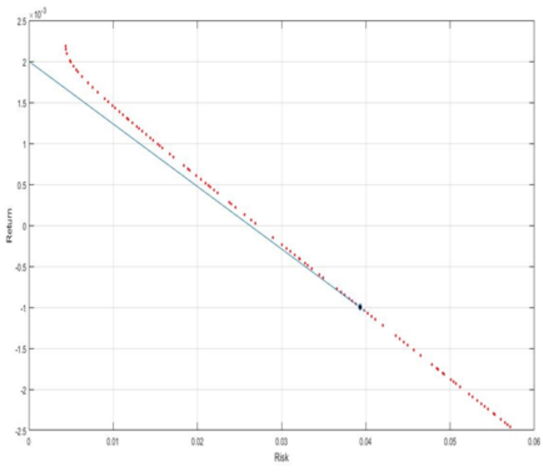

Fig. 7. The efficient boundary of risky purchases with the optimal EAL

The outputs were reported as $E(r_p)^*$=-0.000993 and σ_P^*=0.039 as well as the optimal risky weights on the tangent point W_S^*=0.55 and W_P^*=0.45. Fig. 7 demonstrates the efficient boundary of risky purchases with the optimal EAL. The second step: Given the risk degree with the selection of the best allocation to risky assets *y**, the expected utility function 37 was minimized with respect to constraints and the problem nature. Table 3 demonstrates the optimal values of risky assets (*y**), the complete portfolio variance, the expected return on the complete portfolio, the value of the utility function, the optimal weights of every purchase source, and the expected cost of the complete portfolio of the entire three-month period. According to the results, the optimal ratio of risky assets had inverse relationships with the risk evasion level and risk level measured by variance and it had a direct relationship with the proposed mere risk.

Table 3. The optimal values of risky assets and the expected cost in the complete portfolio during the entire programming horizon

A	y^*	$\sigma[r_c(t)]$	$E[r_c(t)]$	U	W_s	W_p	W_c	Tc (Million Rls)
1	0.88	0.0323	-0.00087	-0.00035	0.484	0.396	0.12	9.969082
1.5	0.6	0.0218	-0.00050	-0.00025	0.330	0.270	0.40	10.180191
2	0.44	0.0145	-0.00033	-0.000099	0.242	0.198	0.56	10.480392
2.5	0.35	0.0126	-0.00022	-0.00005	0.193	0.157	0.64	10.600366
3	0.29	0.0078	-0.00015	0.000056	0.159	0.131	0.710	11.203704
3.5	0.26	0.0075	-0.00009	0.000069	0.143	0.117	0.74	11.622744
4	0.21	0.0071	-0.00005	0.000086	0.116	0.059	0.79	12.120305
5	0.18	0.0070	-0.00001	0.000094	0.083	0.098	0.82	12.739345

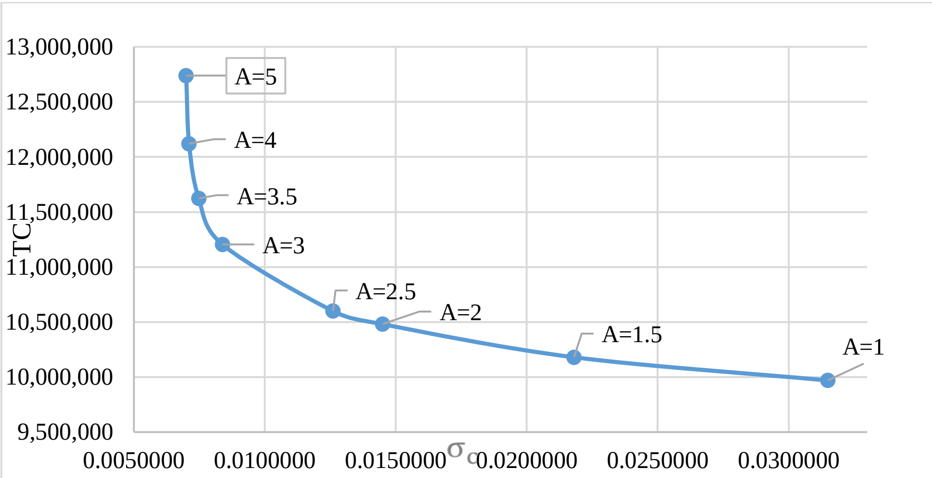

Fig. 8. The expected cost with respect to the standard deviation for different values of A

The risk evasion degree of investor A can now be used to design an optimal risky portfolio and calculate the optimal ratio of energy purchase in the risky component. When the purchaser participates in the market with a risk evasion degree of A=1, the purchase result will account for 88% of the risky component, whereas 12% of energy purchase contracts will be selected. Since the optimal risky portfolio includes 55% of the IRENEX and 45% of the energy pool, the purchase shares of the IRENEX, energy pool, and contracts will be 48.4%, 39%, and 12% in a compete for portfolio. Moreover, the purchase cost will be 9969082 rials for one KW/hr. Fig. 9 indicates the percentages allocated to the risk evasion level of A.

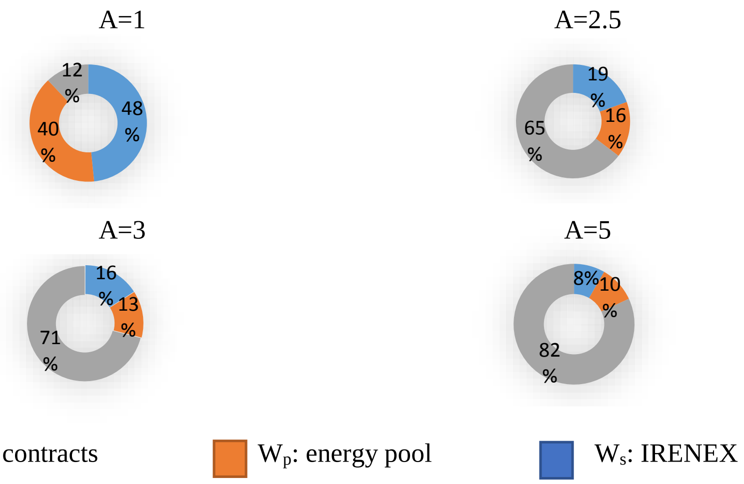

Fig. 9. Allocation of optimal energy purchase portfolio at the risk evasion level of A

When the purchaser participates in the market with a risk evasion level of A= 3, the purchase result will account for 29% and 71% of the risky component and contracts in the complete portfolio. In this case, the expected energy purchase cost is 11203704 rials for one KW/hr, which increased by 12.4% in comparison with the previous case. Fig. 8 indicates the expected cost with respect to the standard deviation for different values of A. When the risk evasion level was A= 5, 18% and 82% of the risky portfolio and contracts were purchased, respectively. In this case, the expected cost increased by 27.8% compared with the risk evasion level of A= 1. According to Table 3 and Fig. 8, changing the purchaser's attitude based on increasing risk evasion decreased the risky component of the portfolio, added to the shares of contracts in the entire portfolio, and increased the expected cost.

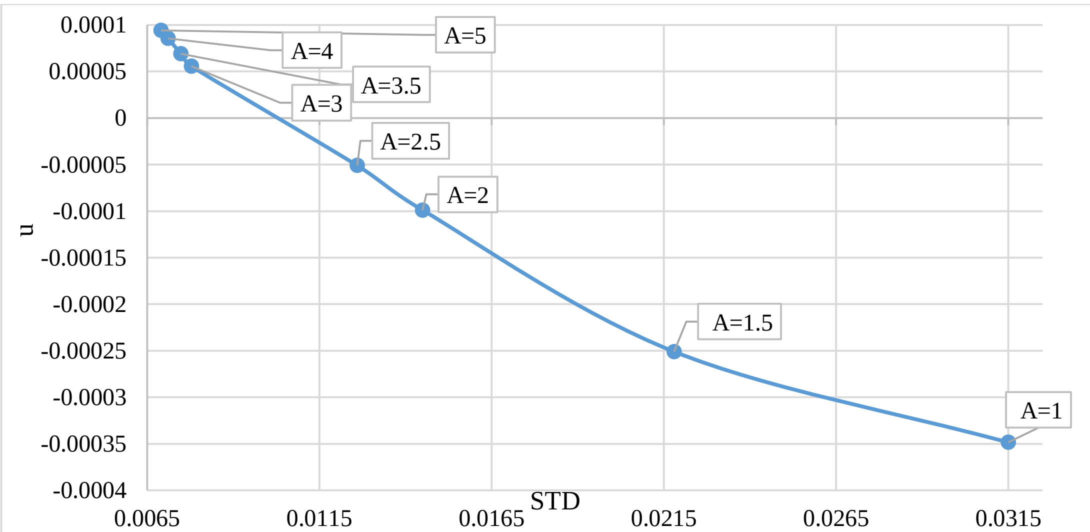

Fig. 10. The value of the preference objective function in comparison with the standard deviation for different values of A

In the preferential objective function (37), the value A of the priority function is severely penalized. As shown in Fig. 10 and Table 3, as the level of risk aversion increased, the value of the objective function increased, which reduced the preferences. Optimizing the allocation of portfolio components between the energy pool and the IRENEX will help improve the reward, *i.e.*, the difference between $E[r_p(t)]\ E(r_B)$ and as well as the portfolio volatility. The EAL resulting from the energy pool, the IRENEX, and contract purchases indicate that the standard deviation of the complete portfolio can reduce. However, the purchase costs increase as the risky optimal portfolio allocation decreases.

Multiobjective Mathematical Model Based on CVaR

After the model was executed by Tool Boxes in MATLAB to determine the optimal portfolio, the collected data were applied to the model. The sensitivity value of γ was analyzed in [0, 10].

Table 4. The expected values of cost, risk, return, and energy allocation in the portfolio for different values of γ

γ	stock	pool	contract	risk	return	Tc(Million Rls)
0	0.481	0.259	0.259	0.059990	-8.72E-05	10.51241100
0.15	0.469	0.266	0.266	0.058324	-2.95E-05	10.54798400
0.25	0.418	0.248	0.334	0.051746	0.000193102	10.68753464
0.5	0.375	0.252	0.373	0.046113	0.000386051	10.83931422
1.25	0.313	0.261	0.426	0.038031	0.000663154	11.27238688
2.5	0.293	0.273	0.434	0.035404	0.000754356	11.41122854
5	0.252	0.286	0.462	0.029918	0.00094329	11.49992654

According to Table 4, it is concluded that a lower ratio of the portfolio is allocated to cash markets, especially the IRENEX, as the energy purchaser becomes more risk-evasive, *i.e.*, the risk evasion coefficient (γ) increases. As γ increases, the expected cost of purchasing energy increases, whereas the risk decreases. Therefore, there is a positive relationship between increasing γ and risk evasion. Fig. 11 demonstrates the efficient boundary diagram based on the expected costs of energy in comparison with the levels of CVaR risks for different values of γ.

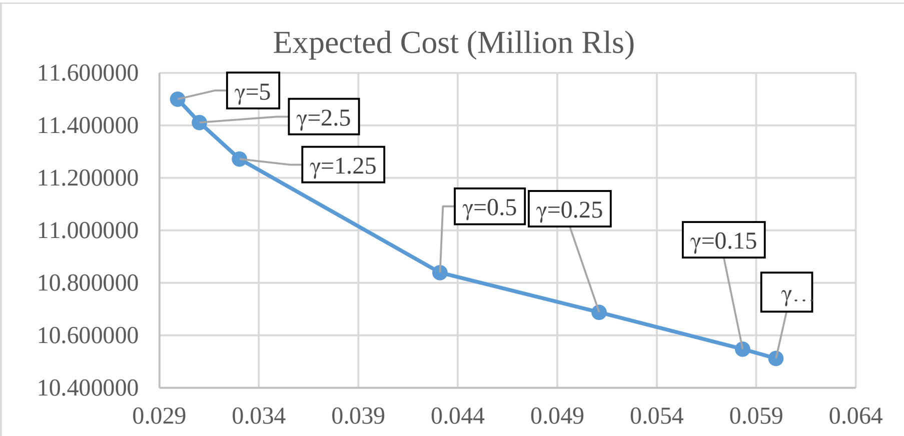

Fig. 11. The expected cost with respect to CVaR for different values of γ

If the risk is ignored, *i.e.*, the value of γ is considered zero, the expected cost of purchasing energy will be calculated as 10565411 rials for one KW/hr. If the value of γ is considered 5 for example, the expected cost will be 11294927 rials for one KW/hr. In this case, the expected cost of purchasing energy increases by 6.9%, whereas the CVaR decreases by 10.1%. In addition, although purchasing energy from bilateral contracts requires higher costs at a high level of risk evasion, it becomes a tool for covering the effective risk by the purchaser due to low volatility and lack of risk. The CVaR and the expected cost will be likely to decrease by 2.9% and 0.2%, respectively. This special scenario is important to the decision-makers who take risks, have no desire to control risk, and like to purchase a portfolio that considers a low level of risk coverage. Based on the expected cost, the CVaR is important to the decision-makers who are risk-evasive and willing to manage risk. According to Fig. 11 and Table 3, the value of γ is placed at the risk evasion level and risk coverage. Given the results of these two models based on the MPT, it can be concluded that the CVaR method provided a more stable energy allocation in electric markets than the mean-variance model, something which is due to the effect of risk evasion. Energy allocation is very unstable in the mean-variance method. In addition, there were no relationships between A and γ. Fig. 12 indicates the allocation of the energy purchase portfolio at the risk evasion level of γ.

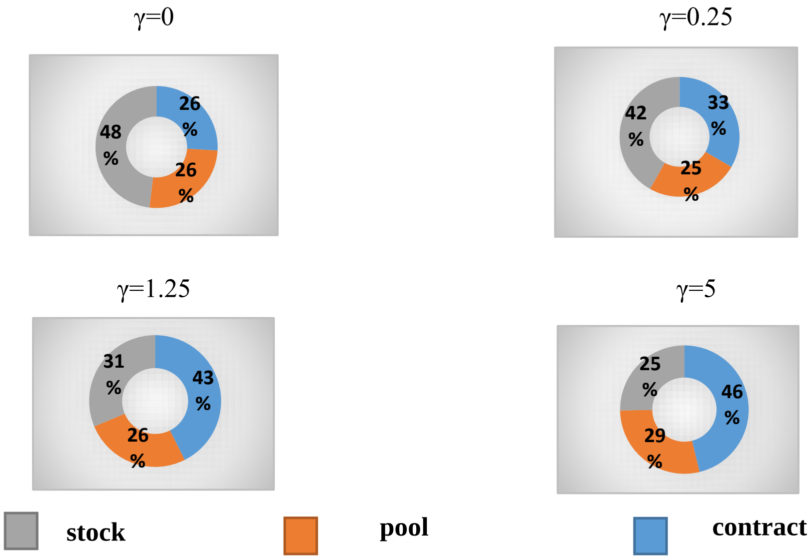

Fig. 12. The optimal energy purchase portfolio allocation at the risk evasion level of γ

Conclusions and suggestions for future studies

Conclusions

This study proposed an efficient approach based on the modern portfolio theory to determine the electrical energy portfolio for big consumers. The proposed approach managed to minimize the purchase costs by considering a level of risk with the best allocation of resources. The proposed models are used in programming problems and proposing electrical energy portfolios. Furthermore, previous studies have mainly focused on the electrical energy sales portfolios, and there are very few studies regarding how to optimize the electrical energy portfolio from the consumer perspective. Considering the programming horizon, this study proposed an applied model for the big consumers of Iran's electrical power industry including bilateral contracts in future risk-free markets as well as the spot-price markets such as the energy pool and the IRENEX to provide the energy carriers. In addition, uncertainty in the spot prices of electrical energy markets of subscribers was taken into account in decision-making. Considering uncertainty can expand the atmosphere of scenarios and complicate decision-making. The artificial neural networks were also employed to predict the future prices of electricity. The MPT-based proposed model benefited from the mean-variance approach and the CVaR method, both of which are employed for financial risk management. The mean-variance model is based on the assumption that the returns follow a normal distribution. The distribution variance can be used as a risk measurement factor. This variance, which is the principle of the mean-variance criterion, was used in the MPT under uncertainty. The risky component of the portfolio decreased as the energy purchaser changed attitudes towards increasing risk evasion, and the shares of contracts increased in the complete portfolio. Moreover, the expected cost also increased in this step, whereas the preference function depended on the risk evasion of the energy purchaser and changed. Optimizing the allocation of portfolio components between the energy pool and the IRENEX will help improve the reward ratio, *i.e.*, the difference between $E[r_p(t)]$ and $E(r_B)$, the volatility of the complete portfolio. According to the EAL resulting from the energy pool, the IRENEX, and the purchase of contracts, the standard deviation of the complete portfolio can decrease; however, the costs of energy purchase increase by decreasing the optimal risky portfolio allocation. As the energy purchaser becomes more risk-evasive in the CVaR model, — *i.e.*, increasing the risk evasion coefficient γ — a lower ratio of the portfolio will be allocated to cash markets, especially the IRENEX. At the same time as γ increases, the expected cost of energy purchase and risk increase, whereas the return decreases. Therefore, there is a positive relationship between increasing the value of γ and risk evasion. According to the two proposed models based on the MPT, it can be concluded that the CVaR provided a more stable energy allocation than the mean-variance model due to the effect of risk evasion. The proposed models indicated that increasing the risk evasion of an energy purchaser resulted in a lower ratio of the portfolio allocated to the risky cash markets due to higher instability. In addition, the preferences and costs of energy purchase decrease and increase, respectively. It should also be added that there are no relationships between A and γ. According to the results, this study provided useful information and acceptable accuracy. The costs of power consumption have significant effects on the net prices of products. Hence, the financial managers and planners of the industries that consume more than 5 MW of electricity per month are advised to design an optimal electrical energy portfolio in electricity markets to minimize costs and reduce the risk for a part of demand.

Suggestions for Future Studies

According to the results, this study provided useful information with high accuracy. The costs of power consumption have substantial effects on the net prices of products. Thus, the financial managers and planners of the industries that consume more than 5 MW of electricity per month are advised to design optimal electrical energy portfolios for cost minimization and risk reduction. Different studies can be conducted and developed in this area. It is recommended that other models and methods be used for the allocation of optimal energy portfolios under uncertainty. The results should then be compared. For instance, fuzzy-random models can be developed instead of using accurate numbers, binary methods, robust optimization, and goal programming in future studies.

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